

**Exercises in GCSE  
Mathematics  
Higher Level  
Robert Joinson**

***Sumbooks***

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## **Exercises in GCSE Mathematics-Higher level**

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# Preface

This book covers the GCSE syllabi examined for the first time in 2003. However, not all parts of the book are used by all the examination boards.

Some areas have more questions than are needed for some pupils. Exercises on pages 1, 8, 9, 10, 11, 14, 18, 19, and 21 contain lots of questions and are aimed at pupils requiring a great deal of practice. These questions are graded and it might only be necessary for some students to do the first column and then each row when they begin to have problems. In general questions in the same row tend to be of the same difficulty, whereas the difficulty increases down the page.

All graphs can be accommodated on A4 size graph paper used in 'portrait' mode.

I would like to thank my wife Jenny and my two daughters Abigail and Hannah for all the help and encouragement they have given me in writing this.

R Joinson

August 2002  
Chester



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## 1. Estimations and Calculations

In questions 1 to 36 use your calculator to work out the value of the problem. In each case

a) write down the calculator display correct to four significant figures

b) write down an estimate you can do to check your answer to part a.

c) write down your answer to part b.

- |  |  |   |
|--|--|---|
| 1) $\frac{1.631 \times 5.482}{4.8143}$                 | 2) $\frac{12.62 \times 3.851}{2.415}$                      | 3) $\frac{9.143 \times 8.639}{4.384}$                       |
| 4) $\frac{0.4831 \times 6.8159}{3.286}$                | 5) $\frac{2.841 \times 0.356}{9.416}$                      | 6) $\frac{8.143 \times 0.3142}{7.8154}$                     |
| 7) $\frac{0.7154 \times 0.832}{5.416}$                 | 8) $\frac{0.1843 \times 0.6315}{6.4153}$                   | 9) $\frac{0.5714 \times 0.8354}{7.831}$                     |
| 10) $\frac{2.453 \times 0.154}{0.3651}$                | 11) $\frac{8.415 \times 0.3142}{0.2164}$                   | 12) $\frac{15.815 \times 0.2231}{0.8145}$                   |
| 13) $\frac{14.264 \times 27.742}{0.6421}$              | 14) $\frac{27.98 \times 96.41}{0.415}$                     | 15) $\frac{184 \times 32.415}{0.283}$                       |
| 16) $\frac{16.854 \times 0.0182}{0.3145}$              | 17) $\frac{27.684 \times 0.8142}{0.0156}$                  | 18) $\frac{67.48 \times 29.684}{0.00142}$                   |
| 19) $\frac{(27.63)^2 \times 14.32}{27.61}$             | 20) $\frac{4.831 \times (0.821)^2}{0.614}$                 | 21) $\frac{23.41 \times (17.83)^2}{126.4}$                  |
| 22) $\frac{(15.84)^2 \times 1.982}{0.614 \times 3.28}$ | 23) $\frac{105 \times (0.325)^2}{0.761 \times 26.45}$      | 24) $\frac{(0.312)^2 \times 27.4}{0.51 \times 6.72}$        |
| 25) $\frac{43.1 \times 6.74}{8.931 + 12.82}$           | 26) $\frac{14.71 \times 23.42}{16.84 + 33.21}$             | 27) $\frac{27.41 \times 0.8541}{4.312 + 6.842}$             |
| 28) $\frac{(26.3)^2 \times 0.416}{2.84(3.14 + 2.86)}$  | 29) $\frac{41.6(8.42 + 3.86)}{0.415}$                      | 30) $\frac{(27.64)^2 \times 19.68}{2.54(2.84 + 6.41)}$      |
| 31) $\frac{27.63 \times (18.41)^2}{0.31(8.62 - 4.31)}$ | 32) $\frac{(28.31)^2 \times (0.843)^2}{0.415}$             | 33) $\frac{36.38 \times (4.321)^2}{0.31(2.865 + 7.416)}$    |
| 34) $\frac{14.841 \times 26.832}{(5.483)^2 + 16.943}$  | 35) $\frac{(41.821)^2 \times 0.0154}{0.831 \times 0.0132}$ | 36) $\frac{(0.531)^2 \times 0.00614}{0.8312 \times 0.0154}$ |

37) If  $v = \frac{(8.64)^2 + 29.83}{0.0154}$  a) Use your calculator to find the value of  $v$ , correct to 3 significant figures. b) What figures would you use to check the value of  $v$ ? c) Write

down the answer to part b.

38) If  $D = \frac{(27.61)^3 \times 0.00814}{3.61(2.48 + 5.61)}$  a) Use your calculator to find the value of  $D$  correct to 4 significant figures. b) What figures would you use to check the value of  $D$ ? c) Write down

the answer to part b.

## 2. Standard Form

### Exercise 1

Write down these numbers in standard form

- |            |               |              |
|------------|---------------|--------------|
| 1) 457     | 2) 1427       | 3) 9431      |
| 4) 156,321 | 5) 17 million | 6) 0.2813    |
| 7) 0.08142 | 8) 0.000486   | 9) 0.0000097 |

### Exercise 2

Change these numbers from standard form

- |                         |                          |                          |
|-------------------------|--------------------------|--------------------------|
| 1) $2.8 \times 10^5$    | 2) $6.4 \times 10^7$     | 3) $9.3 \times 10^4$     |
| 4) $4.315 \times 10^6$  | 5) $8.614 \times 10^9$   | 6) $4.31 \times 10^{-2}$ |
| 7) $3.2 \times 10^{-6}$ | 8) $6.84 \times 10^{-7}$ | 9) $4.38 \times 10^{-9}$ |

### Exercise 3

Calculate each of the following leaving your answers in standard form. Round off to four significant figures wherever necessary.

- |   |  |
|---|--|
| 1) $(6.4 \times 10^2) \times (3.8 \times 10^4)$                               | 2) $(5.4 \times 10^6) \times (8.3 \times 10^2)$                                    |
| 3) $(4.6 \times 10^{-2}) \times (3.4 \times 10^{-3})$                         | 4) $(4.3 \times 10^{-7}) \times (8.8 \times 10^5)$                                 |
| 5) $(8.3 \times 10^{-2}) \times (6.4 \times 10^8)$                            | 6) $(5.3 \times 10^{-7}) \times (4.6 \times 10^3)$                                 |
| 7) $(5.7 \times 10^{-2}) \div (3.4 \times 10^2)$                              | 8) $(8.3 \times 10^{-6}) \div (5.4 \times 10^3)$                                   |
| 9) $(8.4 \times 10^5) \div (2.4 \times 10^{-3})$                              | 10) $(5.4 \times 10^{-7}) \div (4.3 \times 10^{-2})$                               |
| 11) $137,000 \times 10^5$   | 12) $0.08123 \times 10^6$  |
| 13) $27.31 \times 4.82 \times 10^6$   | 14) $571.31 \times 4.2 \times 10^{-7}$   |
| 15) $\frac{3.841 \times 10^6}{3.182 \times 10^2}$                             | 16) $\frac{7.41 \times 10^{-4}}{3.54 \times 10^{-6}}$                              |
| 17) $\frac{(27.41 \times 10^3) \times (2.684 \times 10^7)}{7.41 \times 10^5}$ | 18) $\frac{(2.641 \times 10^{-3}) \times (2.84 \times 10^{-6})}{3.82 \times 10^5}$ |
| 19) $\sqrt{4 \times 10^4}$  | 20) $\sqrt{9 \times 10^{-6}}$  |

### Exercise 4

- The distance from the earth to a star is  $8 \times 10^{13}$  kilometres. Light travels at a speed of approximately  $3.0 \times 10^5$  kilometres per second.
  - How far will light travel in one year?
  - How long will light take to travel from the star to the earth? Give your answer correct to 3 significant figures.
- The mass of the earth is  $5.976 \times 10^{24}$  kilograms and the mass of the moon is  $7.35 \times 10^{22}$  kilograms.
  - Write down these masses in tonnes.
  - How many times is the mass of the earth greater than the mass of the moon?
- A neutron has a mass of  $1.675 \times 10^{-27}$  kilograms and an electron  $9.109 \times 10^{-31}$  kilograms.
  - How many electrons are needed for their mass to be equal to that of a neutron?
  - How many electrons are required to have a mass of 1 kg?



### 3. Rational and Irrational Numbers

#### Exercise 1

Convert the following numbers into fractions

- |                 |                  |                  |                   |
|-----------------|------------------|------------------|-------------------|
| 1) $0.2\dot{5}$ | 2) $0.3\dot{1}$  | 3) $0.5\dot{4}$  | 4) $0.6\dot{2}$   |
| 5) $0.0\dot{4}$ | 6) $0.7\dot{3}$  | 7) $0.00\dot{7}$ | 8) $0.01\dot{7}$  |
| 9) $0.\dot{1}5$ | 10) $0.3\dot{2}$ | 11) $0.\dot{5}3$ | 12) $0.07\dot{2}$ |

#### Exercise 2

Which of the following numbers are rational?

- |                                 |                           |                            |                                  |
|---------------------------------|---------------------------|----------------------------|----------------------------------|
| 1) $\frac{1}{5}$                | 2) $\frac{7}{8}$          | 3) $\frac{1}{\sqrt{2}}$    | 4) $\frac{\sqrt{7}}{\sqrt{8}}$   |
| 5) $\pi$                        | 6) $\pi^2$                | 7) $0.2\dot{3}$            | 8) $\sqrt{3}$                    |
| 9) $\frac{\sqrt{4}}{\sqrt{25}}$ | 10) $\sqrt{\frac{9}{49}}$ | 11) $\frac{\sqrt{5}}{5^2}$ | 12) $\left(\frac{2}{3}\right)^2$ |
| 13) $\sqrt{6.25}$               | 14) $\sqrt{2.5}$          | 15) $\sqrt{4}$             | 16) $\sqrt{4.1}$                 |

#### Exercise 3

Which of the following are irrational?

- |                                |   |  |
|--------------------------------|---|--|
| 1) $1 + \frac{1}{3}$           | 2) $1 + \frac{1}{\sqrt{3}}$               | 3) $1 + \frac{\sqrt{3}}{2}$                    |
| 4) $3 + \frac{1}{\sqrt{3}}$    | 5) $2 - \frac{2}{\sqrt{2}}$               | 6) $\frac{18}{\sqrt{3}} - \frac{12}{\sqrt{3}}$ |
| 7) $2^0 + 2^{-1} + 2^{-2}$     | 8) $(1 + \sqrt{2}) \times (1 + \sqrt{2})$ | 9) $2\sqrt{2}$                                 |
| 10) 2.58                       | 11) $0.3\dot{7}$                          | 12) $1.5\dot{1}$                               |
| 13) $\sqrt{8} \times \sqrt{2}$ | 14) $\sqrt{12} \times \sqrt{2}$           | 15) $\sqrt{2} \times \sqrt{3}$                 |
| 16) $6^{\frac{1}{3}} + 3^0$    | 17) $4^{\frac{1}{3}} \times 9^{-2}$       | 18) $4^{\frac{1}{3}} \times 9^{\frac{1}{3}}$   |

#### Exercise 4

Which of the following equations have rational answers?

- |                                     |                           |                                     |
|-------------------------------------|---------------------------|-------------------------------------|
| 1) $2x = 5$                         | 2) $3x = \sqrt{3}$        | 3) $4x = 2^2$                       |
| 4) $\sqrt{2x} = \frac{\sqrt{2}}{4}$ | 5) $5x = \frac{7}{8}$     | 6) $\sqrt{3x} = \frac{4}{\sqrt{3}}$ |
| 7) $5x^2 = 7$                       | 8) $9x^2 = 4$             | 9) $4x^2 = 17$                      |
| 10) $3x^2 = \frac{1}{2}$            | 11) $\frac{3}{4}x^2 = 12$ | 12) $7x^2 = 5$                      |

#### Exercise 5

Which two of the following are descriptions of irrational numbers?

- A number which, in its decimal form, recurs.
- A number written in its decimal form has a finite number of decimal places.
- A number whose exact value cannot be found.
- A number which can be represented by the ratio of two integers.
- An infinite decimal which does not repeat itself.

## 4. Surds

### Exercise 1

Simplify each of the following by writing as products of whole numbers and surds

- |                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| 1) $\sqrt{8}$    | 2) $\sqrt{12}$   | 3) $\sqrt{24}$   | 4) $\sqrt{28}$   |
| 5) $\sqrt{108}$  | 6) $\sqrt{40}$   | 7) $\sqrt{50}$   | 8) $\sqrt{18}$   |
| 9) $\sqrt{48}$   | 10) $\sqrt{32}$  | 11) $\sqrt{20}$  | 12) $\sqrt{125}$ |
| 13) $\sqrt{200}$ | 14) $\sqrt{216}$ | 15) $\sqrt{192}$ | 16) $\sqrt{320}$ |

### Exercise 2

Simplify

- |                           |                           |                           |                            |
|---------------------------|---------------------------|---------------------------|----------------------------|
| 1) $\frac{2}{\sqrt{2}}$   | 2) $\frac{3}{\sqrt{3}}$   | 3) $\frac{4}{\sqrt{4}}$   | 4) $\frac{6}{\sqrt{2}}$    |
| 5) $\frac{14}{\sqrt{7}}$  | 6) $\frac{8}{\sqrt{2}}$   | 7) $\frac{9}{\sqrt{3}}$   | 8) $\frac{12}{\sqrt{3}}$   |
| 9) $\frac{14}{\sqrt{2}}$  | 10) $\frac{20}{\sqrt{2}}$ | 11) $\frac{30}{\sqrt{3}}$ | 12) $\frac{50}{\sqrt{5}}$  |
| 13) $\frac{70}{\sqrt{5}}$ | 14) $\frac{39}{\sqrt{3}}$ | 15) $\frac{49}{\sqrt{7}}$ | 16) $\frac{63}{\sqrt{21}}$ |

### Exercise 3

Simplify

- |                             |                             |                              |
|-----------------------------|-----------------------------|------------------------------|
| 1) $\sqrt{2} + 2\sqrt{2}$   | 2) $\sqrt{3} + 3\sqrt{3}$   | 3) $2\sqrt{2} + 3\sqrt{2}$   |
| 4) $\sqrt{8} + \sqrt{2}$    | 5) $\sqrt{8} - \sqrt{2}$    | 6) $\sqrt{12} - \sqrt{3}$    |
| 7) $2\sqrt{5} - \sqrt{5}$   | 8) $\sqrt{32} - 2\sqrt{2}$  | 9) $2\sqrt{5} - \sqrt{5}$    |
| 10) $3\sqrt{5} - 2\sqrt{5}$ | 11) $4\sqrt{7} - \sqrt{28}$ | 12) $\sqrt{500} - 3\sqrt{5}$ |

### Exercise 4

Simplify

- |                                       |   |                                       |
|---------------------------------------|---|---------------------------------------|
| 1) $2\sqrt{2} - \frac{2}{\sqrt{2}}$   | 2) $2\sqrt{2} - \frac{3}{\sqrt{2}}$     | 3) $2\sqrt{3} + \frac{3}{\sqrt{3}}$   |
| 4) $\frac{12}{\sqrt{2}} + 2\sqrt{2}$  | 5) $\frac{18}{\sqrt{3}} + 2\sqrt{3}$    | 6) $\frac{28}{\sqrt{7}} - 3\sqrt{7}$  |
| 7) $\frac{30}{\sqrt{5}} + \sqrt{5}$   | 8) $\frac{144}{2\sqrt{3}} - 23\sqrt{3}$ | 9) $\frac{49}{\sqrt{7}} - 3\sqrt{7}$  |
| 10) $\frac{45}{3\sqrt{5}} - \sqrt{5}$ | 11) $\frac{60}{\sqrt{20}} + 3\sqrt{5}$  | 12) $\frac{80}{\sqrt{8}} - 2\sqrt{2}$ |

### Exercise 5

Simplify

- |                                   |                                   |                                   |
|-----------------------------------|-----------------------------------|-----------------------------------|
| 1) $\sqrt{6} \times \sqrt{3}$     | 2) $\sqrt{5} \times \sqrt{10}$    | 3) $\sqrt{3} \times \sqrt{12}$    |
| 4) $\sqrt{6} \times \sqrt{12}$    | 5) $\sqrt{7} \times 2\sqrt{7}$    | 6) $3\sqrt{2} \times 2\sqrt{8}$   |
| 7) $2\sqrt{2} \times 4\sqrt{12}$  | 8) $5\sqrt{6} \times 4\sqrt{3}$   | 9) $4\sqrt{2} \times 3\sqrt{8}$   |
| 10) $5\sqrt{10} \times 2\sqrt{2}$ | 11) $7\sqrt{2} \times 3\sqrt{12}$ | 12) $4\sqrt{8} \times 7\sqrt{12}$ |

## 5. Prime Factors

### Exercise 1

Express the following numbers as products of their prime factors.

- |          |          |          |          |          |
|----------|----------|----------|----------|----------|
| 1) 300   | 2) 900   | 3) 630   | 4) 700   | 5) 792   |
| 6) 945   | 7) 1960  | 8) 1815  | 9) 1512  | 10) 8580 |
| 11) 2640 | 12) 5460 | 13) 3744 | 14) 6336 | 15) 9240 |

### Exercise 2

Express each of the following numbers as products of their prime factors. In each case state the smallest whole number it has to be multiplied by to produce a perfect square.

- |          |          |          |            |            |
|----------|----------|----------|------------|------------|
| 1) 660   | 2) 300   | 3) 450   | 4) 700     | 5) 1575    |
| 6) 2205  | 7) 600   | 8) 396   | 9) 1350    | 10) 1872   |
| 11) 4950 | 12) 3840 | 13) 8820 | 14) 11,760 | 15) 11,340 |

### Exercise 3

Calculate the largest odd number that is a factor of each of the following.

- |          |          |          |          |          |
|----------|----------|----------|----------|----------|
| 1) 120   | 2) 210   | 3) 432   | 4) 416   | 5) 440   |
| 6) 704   | 7) 1144  | 8) 1200  | 9) 1840  | 10) 1848 |
| 11) 2464 | 12) 2112 | 13) 5880 | 14) 4725 | 15) 9240 |

### Exercise 4

1) a) What is the highest common factor of 735 and 756?

An area of land measures 73.5 metres by 75.6 metres. It is to be divided up into square plots of equal size.

b) What is the size of the largest squares that will fit on it?

c) How many squares will fit on it?

2) a) What is the highest common factor of 60 and 75?

b) Jane's mum organises a party for her. She makes 60 cakes and 75 sandwiches. Everyone at the party is allowed the same amount of food to eat. She invites as many children as possible. How many does she invite?

3) a) What is the highest common factor of 990 and 756?

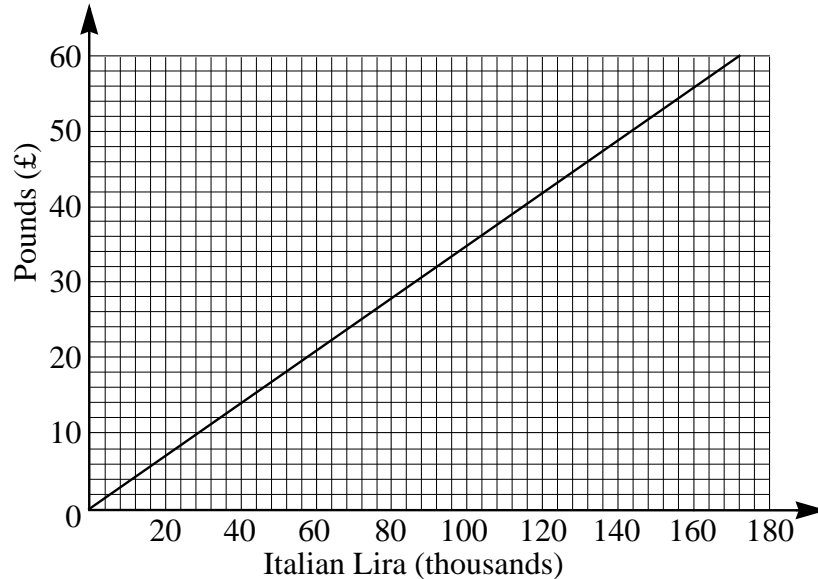
b) A shop is moving its stock. It has 9900 type A items and 7560 type B items. They have to be packed into boxes, each box containing both item A and item B. The same number of type A items and the same number of type B items are in each box. What is the maximum number of boxes needed and c) how many items will be in each box?

## 6. Percentages

- By selling a car for £2,500, John made a profit of 25%. How much did he pay for it?
- A company makes a profit for the year of £75,000 before tax is paid. What percentage tax does it pay if its tax bill amounts to £11,250?
- What is the total cost of a television set if it is priced at £240 plus VAT of  $17\frac{1}{2}\%$ ?
  - A radio costs £21.60 in a sale. If it had a reduction of 10%, what was its original price?
  - A computer costs £998.75 including VAT at  $17\frac{1}{2}\%$ . What is its price before VAT is added?
- Jane invests £1500 in a bank account which pays interest of  $6\frac{1}{2}\%$  per annum.
  - How much interest has she earned at the end of 1 year?
  - She has to pay tax on this interest at 22%. How much tax does she pay?
- Jonathan earns £23,000 per year as a shop manager.
  - If he is offered a pay rise of  $7\frac{1}{2}\%$ , what will his new wages be?
  - Instead he is offered a new job by a different firm and the rate of pay is £25,400 per annum. What percentage increase does this represent on his old wages?
- The population of a certain town was 50,000 at the beginning of 1998. It is expected to rise by 7% each year until the end of the year 2000. What is the expected population at the end of this period?
- A shopkeeper buys 35 radios for £435.75. If she sells them at £15 each, what is her percentage profit?
- A car was bought at the beginning of 1994. During the first year it depreciated in value by 23% and then by 9% each subsequent year. If its original price was £9,000, what was its value at the end of 1997, to the nearest pound?
- A can of cola has a label on it saying '20% extra free'.
  - If the can holds 960ml, what did the original can hold?
  - The original can cost 45p. If the company increase the price of the new can to 60p, does this represent an increase in price? Explain your answer.
- In the general election, Maureen Johnson got 22,016 votes, which was 43% of all the votes cast.
  - If Anthony Jones got 19,968 votes, what percentage of the people voted for him?
  - If John Parry got 8% of the vote, how many people voted for him?
- The cost of building a bridge in 1995 was estimated as £24 million. When it was finally completed in 1998 its total cost amounted to £37.7 million. What was the percentage increase?
- It is estimated that a certain rainforest gets smaller by 8% each year. Approximately how many years will it take to be 39% smaller?
- A firms profits were £500,000 in 1991. In 1992 they were 15% higher. However in 1993 they were 5% lower than in the previous year. What were the profits in 1993?
- £1000 is invested at 6% compound interest. Interest is added to the investment at the end of each year. For how many years must the money be invested to in order to get at least £400 interest?
- A lady wants a room to be built onto her house. Builder A quotes £11,400 which includes VAT of  $17\frac{1}{2}\%$ . However he will reduce this by 10% if it is accepted within one week.  
Builder B quotes £9000 excluding VAT. Which is the cheapest quotation and by how much?
- VAT of  $17\frac{1}{2}\%$  is added to the cost of a computer. If the VAT is £166.25, what is the total cost of the computer?

## 7. Conversion Graphs

1. This graph shows the relationship between the Italian lira and the pound sterling.



From the graph determine:

- The rate of exchange (i.e. the number of lira to the pound)
  - The number of lira that can be obtained for £22.
  - The amount of £ sterling that can be exchanged for 140,000 lira.
2. Christmas trees are priced according to their height. The table below shows some of the prices charged last Christmas.

Height (metres)	1.0	1.5	2.0	2.5	3.0
Price (£)	8.50	11.75	15.00	18.25	21.50

Draw a conversion graph using 8cm to represent 1 metre on the vertical axis and 1cm to represent £2 on the horizontal axis.

- From your graph, calculate the cost of a tree measuring 2m 35cm.
  - What size tree can be purchased for £12.50?
3. Bill is a craftsman who makes wooden bowls on his lathe. He advertises that he can make any size bowl between 20cm and 60cm diameter. In his shop he gives the price of five different bowls as an example.

Diameter of bowl (cm)	20	30	40	50	60
Price	£8.80	£20.00	£42.40	£79.00	£133.60

- Use these figures to draw a conversion graph. Use a scale of 2cm to represent a diameter of 10cm on the horizontal axis and 2cm to represent £10 on the vertical axis.
- Jane has £50 to spend. From your diagram, estimate the size of bowl she can buy.
- What is the cost of a bowl of 34cm diameter?

## 8. Fractions

### Exercise 1

Simplify into single fractions

- |                                    |                                   |                                    |                                    |
|------------------------------------|-----------------------------------|------------------------------------|------------------------------------|
| 1) $\frac{1}{5} + \frac{2}{5}$     | 2) $\frac{1}{3} + \frac{2}{3}$    | 3) $\frac{7}{10} + \frac{7}{10}$   | 4) $\frac{1}{5} + \frac{1}{3}$     |
| 5) $\frac{1}{8} + \frac{1}{7}$     | 6) $\frac{1}{3} - \frac{1}{4}$    | 7) $\frac{3}{5} + \frac{2}{3}$     | 8) $\frac{4}{5} - \frac{2}{7}$     |
| 9) $\frac{7}{9} + \frac{7}{12}$    | 10) $1\frac{3}{4} - \frac{3}{5}$  | 11) $2\frac{7}{8} - \frac{11}{12}$ | 12) $4\frac{3}{7} - \frac{4}{5}$   |
| 13) $\frac{a}{7} + \frac{2a}{7}$   | 14) $\frac{4a}{5} + \frac{2a}{5}$ | 15) $\frac{a}{4} + \frac{3a}{4}$   | 16) $\frac{x}{5} + \frac{x}{3}$    |
| 17) $\frac{x}{5} + \frac{x}{7}$    | 18) $\frac{b}{2} - \frac{b}{5}$   | 19) $\frac{3x}{8} - \frac{x}{4}$   | 20) $\frac{7x}{10} - \frac{2x}{7}$ |
| 21) $\frac{3x}{10} + \frac{4x}{9}$ | 22) $\frac{3a}{2} + \frac{3a}{4}$ | 23) $\frac{15x}{7} + \frac{x}{2}$  | 24) $\frac{11x}{4} - \frac{x}{11}$ |

### Exercise 2

Simplify

- |                                  |                                  |                                  |                                  |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1) $\frac{2}{x} + \frac{3}{x}$   | 2) $\frac{7}{a} - \frac{6}{a}$   | 3) $\frac{6}{x} + \frac{9}{x}$   | 4) $\frac{1}{x} + \frac{3}{2x}$  |
| 5) $\frac{5}{2x} + \frac{3}{2x}$ | 6) $\frac{3}{4x} - \frac{1}{5x}$ | 7) $\frac{2}{3x} + \frac{3}{2x}$ | 8) $\frac{5}{4a} - \frac{4}{5a}$ |
| 9) $\frac{1}{a} + \frac{1}{b}$   | 10) $\frac{2}{x} - \frac{1}{y}$  | 11) $\frac{3}{x} + \frac{5}{y}$  | 12) $\frac{1}{5} + \frac{1}{b}$  |
| 13) $\frac{2}{5} + \frac{3}{b}$  | 14) $\frac{4}{5} + \frac{x}{5}$  | 15) $\frac{4}{x} - \frac{x}{4}$  | 16) $\frac{x}{4} - \frac{3}{x}$  |

### Exercise 3

Simplify

- |   |   |   |
|---|---|---|
| 1) $\frac{3}{4} + \frac{2}{a+1}$          | 2) $\frac{5}{8} - \frac{1}{x+1}$          | 3) $\frac{3}{10} - \frac{2}{x-1}$         |
| 4) $\frac{x}{3} + \frac{x-1}{4}$          | 5) $\frac{a}{4} - \frac{x+1}{3}$          | 6) $\frac{2b}{5} + \frac{3a}{4}$          |
| 7) $\frac{4x+3}{9} + \frac{5x}{2}$        | 8) $\frac{5a}{4} + \frac{2a+3}{5}$        | 9) $\frac{7x+4}{3} - \frac{5x}{8}$        |
| 10) $\frac{x}{2} - \frac{2x-1}{3}$        | 11) $\frac{4y+3}{6} - \frac{y}{5}$        | 12) $\frac{7a}{4} + \frac{3a+5}{7}$       |
| 13) $\frac{6}{x-1} + \frac{3}{x-4}$       | 14) $\frac{7}{x+2} + \frac{4}{2x-1}$      | 15) $\frac{5}{3x+2} - \frac{2}{4x+1}$     |
| 16) $\frac{5}{2x} + \frac{y}{3}$          | 17) $\frac{3x}{a} + \frac{4x}{5a}$        | 18) $\frac{4x}{a} - \frac{5x}{3a}$        |
| 19) $\frac{1}{5}(x+1) - \frac{1}{6}(x+3)$ | 20) $\frac{5}{8}(x+1) - \frac{2}{5}(x-3)$ | 21) $\frac{3}{5}(a-2) + \frac{3}{7}(a-1)$ |
| 22) $\frac{3(x+3)}{4} + x$                | 23) $\frac{5(a+6)}{4} - a$                | 24) $\frac{11(x+3)}{4} - 2x$              |
| 25) $x + \frac{2}{x}$                     | 26) $3a + \frac{4a+1}{3}$                 | 27) $5x - \frac{3x+3}{4}$                 |

## 9. Indices

### Exercise 1

Simplify each of the following

- |                               |                                 |                                |                                  |
|-------------------------------|---------------------------------|--------------------------------|----------------------------------|
| 1) $x^2 \times x^5$           | 2) $y^4 \times y^7$             | 3) $a^3 \times a^4$            | 4) $b^9 \times b^3$              |
| 5) $3x^2 \times 4$            | 6) $5a^3 \times a^4$            | 7) $6y^2 \times 4y^3$          | 8) $7x^4 \times 5x^7$            |
| 9) $a^5 \div a^2$             | 10) $b^7 \div b^3$              | 11) $y^6 \div y^4$             | 12) $y^7 \div y^2$               |
| 13) $(a^2)^3$                 | 14) $(c^3)^6$                   | 15) $(x^7)^4$                  | 16) $(x^3)^4$                    |
| 17) $x^2y \times xy^2$        | 18) $a^3b \times a^4b^5$        | 19) $x^4y^5 \times y^3$        | 20) $a^3b^3 \times a^4b^4$       |
| 21) $(4x)^2$                  | 22) $(3x^2)^2$                  | 23) $(4y^3)^2$                 | 24) $(2a^2)^4$                   |
| 25) $12a^2 \div 4$            | 26) $18x^5 \div x^2$            | 27) $16y^5 \div 4y^2$          | 28) $20a^5 \div 5a^2$            |
| 29) $\frac{24a^4}{6a^2}$      | 30) $\frac{18b^7}{3b}$          | 31) $\frac{12a^3b^2}{3ab}$     | 32) $\frac{21x^3y^5}{7x^2y^4}$   |
| 33) $x^5 \times x^2 \div x^3$ | 34) $3a^2 \times 2a \times a^5$ | 35) $4y^2 \times 7y^3 \div 2y$ | 36) $4x^7 \times 3x^4 \div 4x^3$ |

### Exercise 2

Simplify

- |  |  |   |   |
|--|--|---|---|
| 1) $x^9 \div x^9$                            | 2) $a^5 \div a^7$                            | 3) $20y^3 \div 10y^4$                       | 4) $4b^3 \div 8b^5$                           |
| 5) $y^2 \times y^{-3}$                       | 6) $2x^2 \times 3x^{-5}$                     | 7) $4a^3 \times 3a^{-6}$                    | 8) $3a^2b \times 2ab^{-2}$                    |
| 9) $x^3 \times x^0$                          | 10) $y^{-3} \times y^0$                      | 11) $3x^2 \times 2x^0$                      | 12) $6a^0 \times 4a^0$                        |
| 13) $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ | 14) $x^{\frac{1}{3}} \times x^{\frac{1}{3}}$ | 15) $b^0 \times b^{\frac{1}{2}}$            | 16) $y^{\frac{1}{2}} \times y^{-\frac{3}{2}}$ |
| 17) $x^{\frac{1}{2}} \div x^{\frac{1}{2}}$   | 18) $y^{\frac{1}{2}} \div y^{\frac{1}{4}}$   | 19) $a^{\frac{1}{4}} \div a^{-\frac{1}{2}}$ | 20) $b^{\frac{1}{2}} \div b^0$                |
| 21) $(x^{\frac{1}{2}})^3$                    | 22) $(a^{\frac{1}{4}})^2$                    | 23) $(2b^{\frac{1}{3}})^3$                  | 24) $(5y^3)^{\frac{1}{2}}$                    |

### Exercise 3

Simplify

- |                          |                          |                                    |                                     |
|--------------------------|--------------------------|------------------------------------|-------------------------------------|
| 1) $25^{\frac{1}{2}}$    | 2) $25^{-\frac{1}{2}}$   | 3) $8^{\frac{1}{3}}$               | 4) $27^{\frac{1}{3}}$               |
| 5) $8^{\frac{2}{3}}$     | 6) $4^{\frac{3}{2}}$     | 7) $(4^{\frac{1}{3}})^3$           | 8) $9^{1\frac{1}{2}}$               |
| 9) $(81)^{\frac{3}{4}}$  | 10) $(64)^{\frac{2}{3}}$ | 11) $(32)^{\frac{3}{5}}$           | 12) $25^{\frac{3}{2}}$              |
| 13) $(25)^{\frac{3}{2}}$ | 14) $(64)^{\frac{5}{6}}$ | 15) $(6\frac{1}{4})^{\frac{1}{2}}$ | 16) $(\frac{125}{8})^{\frac{2}{3}}$ |

### Exercise 4

Solve the following equations

- |                                     |                                     |                                      |                              |
|-------------------------------------|-------------------------------------|--------------------------------------|------------------------------|
| 1) $36^{\frac{1}{x}} = 6$           | 2) $81^{\frac{1}{x}} = 9$           | 3) $81^{\frac{1}{x}} = 3$            | 4) $64^{\frac{1}{x}} = 4$    |
| 5) $32^x = 2$                       | 6) $25^x = \frac{1}{5}$             | 7) $81^{\frac{1}{2}} = 3^x$          | 8) $512^{\frac{1}{3}} = 2^x$ |
| 9) $32^{\frac{1}{x}} = \frac{1}{2}$ | 10) $x^{\frac{1}{4}} = \frac{1}{3}$ | 11) $x^{-\frac{1}{4}} = \frac{1}{2}$ | 12) $16^x = \frac{1}{4}$     |

## 10. Simplifying

### Exercise 1

Simplify each of the following by expanding the brackets where necessary

- |                                   |                                       |
|-----------------------------------|---------------------------------------|
| 1) $7x - 3x$                      | 2) $5y - 7y$                          |
| 3) $-8y + 3y$                     | 4) $-6x - 7x$                         |
| 5) $4x + 3y + 5x + 6y$            | 6) $9y + 7x - 11y - 4x$               |
| 7) $-6x + 3y - 4x + 2y$           | 8) $-7x - 6y + 3x - 4y$               |
| 9) $2ab + 3b - 4a - 6ab$          | 10) $12b - 4a + 3ab - 7a$             |
| 11) $4x^2 + 3x - 2x^2$            | 12) $6y^2 - 4y - 5y^2$                |
| 13) $4y^2 - 4xy + 4xy$            | 14) $-6x^2 + 3y^2 - 4x^2$             |
| 15) $3(x + y) + y$                | 16) $5(2x - y) + 2y$                  |
| 17) $3x + 4(x + y)$               | 18) $7x - 3(x + 2y)$                  |
| 19) $4y - 2(x - y)$               | 20) $6x - 4(2x - 2y)$                 |
| 21) $3(x + y) + 2(x - y)$         | 22) $4(2x + 3y) - (x + 2y)$           |
| 23) $5(2x - 3y) - (2x - y)$       | 24) $3(2x - y) - 2(3x - y)$           |
| 25) $3(2x - 3) - 3(x - 4)$        | 26) $5(2x - 3y) - 3(5x - 2y)$         |
| 27) $4x(2x - 3) - 3x(2x + 4)$     | 28) $7x(y - 2) - 6y(2x - 3)$          |
| 29) $4y(2y - 3x) - 2x(x - 3y)$    | 30) $4x(4y + 3x) - 3y(4x - 3y)$       |
| 31) $\frac{1}{4}x + \frac{1}{3}x$ | 32) $\frac{1}{2}x - \frac{1}{3}x$     |
| 33) $\frac{2}{3}y - \frac{1}{2}y$ | 34) $\frac{1}{3}y^2 + \frac{1}{6}y^2$ |

### Exercise 2

Expand and simplify

- |                        |                          |
|------------------------|--------------------------|
| 1) $(x + 1)(x - 3)$    | 2) $(x + 2)(x - 4)$      |
| 3) $(2x + 3)(x - 7)$   | 4) $(2x + 5)(3x - 2)$    |
| 5) $(2x - 7)(3x + 2)$  | 6) $(3x + 4)(2x - 5)$    |
| 7) $(5x + 2)(7x - 3)$  | 8) $(4x + 8)(3x - 2)$    |
| 9) $(4x - 4)(2x + 1)$  | 10) $(3x + 2y)(2x + 3y)$ |
| 11) $(4x + y)(x + 3y)$ | 12) $(x - 4)(2x - 6)$    |
| 13) $(x + 1)^2$        | 14) $(3x + 2)^2$         |
| 15) $(5x - 2)^2$       | 16) $(3x + 4)^2$         |
| 17) $(5x - 6)^2$       | 18) $(7x + 2)^2$         |
| 19) $(-3x + 2)^2$      | 20) $(-5x - 7)^2$        |
| 21) $(-4x - 6)^2$      | 22) $(x + y)^2$          |
| 23) $(2x + 3y)^2$      | 24) $(4x - 2y)^2$        |



## 11. Re-arranging Formulae

### Exercise 1

In each of the following questions, re-arrange the equation to make the letter in the bracket the subject.

- |                                      |     |                                      |     |
|--------------------------------------|-----|--------------------------------------|-----|
| 1) $v = u + at$                      | (u) | 2) $v = u + at$                      | (a) |
| 3) $d = 3b - c$                      | (b) | 4) $c = pd + w$                      | (w) |
| 5) $x = 7y - z$                      | (z) | 6) $a = 3b + c$                      | (b) |
| 7) $w = \frac{4v + u}{3}$            | (v) | 8) $x = \frac{5y + b}{4}$            | (b) |
| 9) $2x = x + b$                      | (b) | 10) $6y = 3a - 2y$                   | (a) |
| 11) $p = \frac{1}{2}a + 3b$          | (a) | 12) $w = 2v + \frac{1}{4}u$          | (v) |
| 13) $c = \frac{a + b}{d}$            | (d) | 14) $p = \frac{2r - q}{3s}$          | (s) |
| 15) $x = 2(y + z)$                   | (z) | 16) $a = 3(3b + 4c)$                 | (c) |
| 17) $x = \frac{1}{2}(y + z)$         | (y) | 18) $3a = \frac{1}{3}(2b + c)$       | (c) |
| 19) $3x = \frac{1}{4}(y - z)$        | (z) | 20) $5w = \frac{1}{3}(3v - 2u)$      | (u) |
| 21) $7x - 4y = \frac{1}{2}(3x + 6y)$ | (x) | 22) $5a + 3b = \frac{2}{3}(3b - 2a)$ | (a) |

### Exercise 2

In each of the following questions, re-arrange the equation to make the letter in the bracket the subject.

- |  |     |   |     |
|--|-----|---|-----|
| 1) $a = \frac{b^2}{c}$                                 | (b) | 2) $x = \frac{y}{z^2}$                                | (z) |
| 3) $c = \frac{4a^2}{b}$                                | (a) | 4) $3v = \frac{9}{u^2}$                               | (u) |
| 5) $\frac{1}{2}x = \frac{2}{3}x + y$                   | (y) | 6) $\frac{3}{4}y - 2x = y$                            | (y) |
| 7) $\frac{7}{2}x = \frac{1}{2}(x + y)$                 | (x) | 8) $\frac{4}{9}b = \frac{1}{4}(b - 3c)$               | (b) |
| 9) $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$           | (x) | 10) $\frac{2}{3x} = \frac{2}{y} + \frac{3}{z}$        | (x) |
| 11) $\frac{1}{\sqrt{x}} = \frac{1}{2a} + \frac{1}{3b}$ | (x) | 12) $\frac{2}{\sqrt{x}} = \frac{3}{2y} + \frac{b}{2}$ | (x) |
| 13) $\frac{2}{3x} = \frac{y}{2} - \frac{1}{z}$         | (z) | 14) $\frac{3}{x} = \frac{6}{y} - \frac{1}{z}$         | (z) |
| 15) $x = \frac{1}{a^2} + \frac{1}{b}$                  | (a) | 16) $4y = \frac{2}{3a^2} + 3b$                        | (a) |
| 17) $\frac{3}{b} = \frac{1}{b} + \frac{1}{c}$          | (b) | 18) $\frac{3}{x} - \frac{6}{y} = \frac{1}{x}$         | (x) |
| 19) $3x = \frac{2y + z}{y}$                            | (y) | 20) $4a = \frac{b - 3c}{c}$                           | (c) |
| 21) $\frac{1}{x} = \frac{x + 3y}{x}$                   | (x) | 22) $\frac{2}{y} = \frac{x - 3y}{2y}$                 | (y) |

## 12. Number Sequences

### Exercise 1

Write down the next two numbers in the following sequences

- |                                  |                                  |
|----------------------------------|----------------------------------|
| 1) 2, 5, 8, 11, 14, 17 ...       | 2) 1, 5, 9, 13, 17, 21 ...       |
| 3) 4, 5, 7, 10, 14, 19 ...       | 4) 2, 2, 3, 5, 8, 12 ...         |
| 5) 17, 19, 22, 26, 31, 37 ...    | 6) 20, 19, 17, 14, 10, 5 ...     |
| 7) 10, 8, 6, 4, 2, 0 ...         | 8) 15, 14, 11, 6, -1, -10 ...    |
| 9) 6, 7, 7, 6, 4, 1 ...          | 10) 2, 4, 8, 16, 32, 64 ...      |
| 11) -1, -2, -4, -7, -11, -16 ... | 12) -6, -6, -7, -9, -12, -16 ... |
| 13) -7, -1, 5, 11, 17, 23 ...    | 14) -3, -1, 0, 0, -1, -3 ...     |
| 15) 1, 3, 7, 15, 31, 63 ...      | 16) 1, 4, 9, 16, 25, 36 ...      |
| 17) 7, 5, 3, 1, -1, -3 ...       | 18) 1, 8, 27, 64, 125, 216 ...   |
| 19) -7, 0, 19, 56, 117, 208 ...  | 20) -2, 1, 6, 13, 22, 33 ...     |
| 21) 1, 2, 4, 8, 16, 32 ...       | 22) 2, 11, 26, 47, 74, 107 ...   |
| 23) 0, 1, 1, 2, 3, 5 ...         | 24) 6, 7, 13, 20, 33, 53 ...     |
| 25) 1, 3, 6, 10, 15, 21 ...      | 26) 4, 9, 15, 22, 30, 39 ...     |

### Exercise 2

Write down the next two numbers and find the rule, in terms of the  $n^{\text{th}}$  number, for each of the following sequences

- |  |  |
|--|--|
| 1) 4, 7, 10, 13, 16, 19 ...  | 2) -3, 2, 7, 12, 17, 22 ...  |
| 3) -2, -6, -10, -14, -18, -22 ...  | 4) 8, 3, -2, -7, -12, -17 ...  |
| 5) 39, 31, 23, 15, 7, -1 ...   | 6) 6, 11, 16, 21, 26, 31 ...   |
| 7) -19, -12, -5, 2, 9, 16 ...  | 8) 1, 4, 9, 16, 25, 36 ...   |
| 9) 3, 6, 11, 18, 27, 38, ...   | 10) -6, -3, 2, 9, 18, 29, ...  |
| 11) 0, 1, 4, 9, 16, 25 ...   | 12) 0, 2, 6, 12, 20, 30 ...  |
| 13) $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \frac{10}{11}, \frac{12}{13}$ | 14) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$ |
| 15) $\frac{1}{4}, \frac{4}{5}, \frac{9}{6}, \frac{16}{7}, \frac{25}{8}, 4$             | 16) $3, \frac{6}{7}, \frac{9}{17}, \frac{12}{31}, \frac{15}{49}, \frac{18}{71}$    |

### Exercise 3

- 1) a) Write down an expression for the  $n^{\text{th}}$  term of the sequence 2, 3, 4, 5, 6 ...  
 b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.
- 2) a) Write down an expression for the  $n^{\text{th}}$  term of the sequence 3, 5, 7, 9, 11 ...  
 b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.
- 3) a) Write down an expression for the  $n^{\text{th}}$  term of the sequence 1, 4, 9, 16, 25, 36 ...  
 b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.
- 4) a) Write down an expression for the  $n^{\text{th}}$  term of the sequence 5, 9, 13, 17, 21, 27 ...  
 b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.

### 13. Substitution

#### Exercise 1

Evaluate each of the following, given that  $a = 6$ ,  $b = 4$  and  $c = -5$ .

- |                  |                   |                 |                   |
|------------------|-------------------|-----------------|-------------------|
| 1) $3a + 5b$     | 2) $4a - 6b$      | 3) $2a - 7b$    | 4) $4c + 2a$      |
| 5) $5a - 4c$     | 6) $3a^2 + 2b$    | 7) $4b^2 - 2a$  | 8) $5a + 3c^2$    |
| 9) $(4b - 6c)^2$ | 10) $(2a)^2 + 3b$ | 11) $4c - 5b^2$ | 12) $6a - (5c)^2$ |

#### Exercise 2

- If  $y = 3x + 4$ 
  - Calculate the value of  $y$  when  $x = 6$
  - What value of  $x$  is needed if  $y = 19$ ?
- If  $a = 3b - 6$ 
  - Calculate the value of  $a$  when  $b = -7$
  - What value of  $b$  is needed if  $a = 54$ ?
- If  $y = 7x + 3$ 
  - Calculate the value of  $y$  when  $x = -6$
  - What value of  $x$  is needed if  $y = -81$ ?
- Carol works for a garden centre and plants rose bushes in her nursery. She works out the length of each row of bushes using the formula  $L = 50R + 200$ , where  $L$  represents the length of the row in centimetres and  $R$  is the number of bushes she plants. Use the formula to calculate
  - the length of a row containing 10 bushes
  - the number of bushes in a row 15 metres long.
- The volume of a cone is given by the formula  $V = \frac{1}{3}\pi r^2 h$ .
  - Calculate the volume of a cone when  $\pi = 3.142$ ,  $r = 3\text{cm}$  and  $h = 2.5\text{cm}$ .
  - A cone has a volume of  $183\text{cm}^3$ . Calculate the value of its height  $h$ , if  $r = 5\text{cm}$  and  $\pi = 3.142$ . Give your answer correct to the nearest millimetre.
- Simple interest can be calculated from the formula  $I = \frac{PTR}{100}$ .
  - If the principal ( $P$ ) = £250, the time ( $T$ ) = 3 years and the rate ( $R$ ) = 9.5%, calculate the interest.
  - If the interest required is £200, what principal needs to be invested for 6 years at 7%?
- The temperature  $F$  (degrees fahrenheit) is connected to the temperature  $C$  (degrees celsius) by the formula  $C = \frac{5}{9}(F - 32)$ .
  - Calculate  $C$  if  $F = -20^\circ F$ .
  - Convert  $-10^\circ C$  into  $^\circ F$
- A bus company uses the formula  $T = \frac{D}{20} + \frac{S}{60} + \frac{1}{4}$  to calculate the time needed for their bus journeys.  $D$  is the distance in miles and  $S$  the number of stops on the journey. If  $T$  is measured in hours, calculate
  - the time needed for a bus journey of 10 miles with 10 stops.
  - the time needed for a bus journey of 20 miles with 4 stops.
  - During the rush hour, more people get on the bus and the extra traffic slows the bus down. What would you do to the formula to take this into account?

## 14. Factorising

### Exercise 1

Factorise each of the following

- |                                     |                                    |                                     |
|-------------------------------------|------------------------------------|-------------------------------------|
| 1) $4x + 8$                         | 2) $6y - 9$                        | 3) $7b - 14a$                       |
| 4) $xy - x$                         | 5) $xy + 3x$                       | 6) $4y + 10xy$                      |
| 7) $6x^2 + 2$                       | 8) $5x^2 - x$                      | 9) $9x^2 - 3x$                      |
| 10) $a^2b + ab^2$                   | 11) $4ab - a^2b$                   | 12) $8ab + 6ab^2$                   |
| 13) $a + ab - a^2$                  | 14) $3ab - ac + a^2$               | 15) $5x^2y - 4y^2 - 3xy$            |
| 16) $\frac{x^2}{2} - \frac{x^3}{4}$ | 17) $\frac{y^2}{3} - \frac{xy}{6}$ | 18) $\frac{5x^2}{6} - \frac{2x}{3}$ |

### Exercise 2

Factorise

- |                    |                     |                      |
|--------------------|---------------------|----------------------|
| 1) $m^2 - n^2$     | 2) $a^2 - 4$        | 3) $(xy)^2 - z^2$    |
| 4) $(ab)^2 - 9$    | 5) $x^2y^2 - 4$     | 6) $v^2w^2 - 25$     |
| 7) $a^2b^2 - 9c^2$ | 8) $25a^2 - 9b^2$   | 9) $b^2 - 1$         |
| 10) $2a^2 - 50$    | 11) $8a^2 - 50$     | 12) $12x^2 - 27y^2$  |
| 13) $xy^2 - 4x^3$  | 14) $2xy^2 - 8x^3$  | 15) $4x^2y^2 - 9x^2$ |
| 16) $x^4 - y^4$    | 17) $16x^4 - 81y^4$ | 18) $3a^4 - 12b^2$   |

### Exercise 3

Factorise

- |                      |                     |                      |
|----------------------|---------------------|----------------------|
| 1) $x^2 + 4x + 3$    | 2) $x^2 + 4x + 4$   | 3) $x^2 + 8x + 7$    |
| 4) $x^2 + 7x + 10$   | 5) $x^2 + 7x + 12$  | 6) $x^2 + 11x + 30$  |
| 7) $x^2 + 2x - 3$    | 8) $x^2 - 2x + 3$   | 9) $x^2 + 4x - 5$    |
| 10) $x^2 - 2x - 8$   | 11) $x^2 - 2x - 15$ | 12) $x^2 - x - 12$   |
| 13) $x^2 - 10x + 24$ | 14) $x^2 - 8x + 15$ | 15) $x^2 - 11x + 28$ |

### Exercise 4

Factorise

- |                       |                       |                       |
|-----------------------|-----------------------|-----------------------|
| 1) $2x^2 + 3x + 1$    | 2) $2x^2 + 9x + 4$    | 3) $2x^2 + 7x + 3$    |
| 4) $2x^2 + 8x + 6$    | 5) $2x^2 + x - 6$     | 6) $3x^2 - 7x - 6$    |
| 7) $2x^2 - 9x + 4$    | 8) $3x^2 - 10x + 3$   | 9) $3x^2 - 14x + 8$   |
| 10) $3x^2 + x - 14$   | 11) $3x^2 + 19x + 20$ | 12) $3x^2 - 12x + 12$ |
| 13) $4x^2 + 10x + 6$  | 14) $4x^2 - 10x + 6$  | 15) $4x^2 + 13x + 3$  |
| 16) $4x^2 + 21x + 5$  | 17) $5x^2 + 13x - 6$  | 18) $6x^2 - 5x - 6$   |
| 19) $6x^2 + 5x + 1$   | 20) $9x^2 + 12x + 4$  | 21) $8x^2 + 11x + 3$  |
| 22) $4x^2 - 23x + 15$ | 23) $5x^2 - 13x - 6$  | 24) $12x^2 - 13x + 3$ |

## 15. Trial and Improvement

### Exercise 1

By using the method of trial and improvement, calculate the value of  $x$  in each of the following equations. In each case show all your workings and give each answer correct to 1 decimal place.

- |                      |                      |                     |
|----------------------|----------------------|---------------------|
| 1) $x^3 = 53$        | 2) $x^3 = 77$        | 3) $x^3 = 101$      |
| 4) $x^2 + x = 36$    | 5) $2x^2 + x = 41$   | 6) $x^2 + 3x = 61$  |
| 7) $5x^2 - 3x = 120$ | 8) $6x^2 - 2x = 77$  | 9) $3x^2 - 5x = 5$  |
| 10) $x^3 + 3x = 16$  | 11) $x^3 - 3x = 96$  | 12) $x^3 - 4x = 12$ |
| 13) $2x^3 + 4x = 96$ | 14) $3x^3 - 4x = 14$ | 15) $2x^3 - 5x = 4$ |

### Exercise 2

- 1) A square has an area of  $45\text{cm}^2$ . Using the method of trial and improvement, calculate the length of its sides, correct to one decimal place. Show all your calculations.
- 2) A rectangle has one side 4cm longer than the other. If its area is  $100\text{cm}^2$ , use the method of trial and improvement to calculate the length of the shorter side, correct to 1 decimal place. Show all your calculations.
- 3) The perpendicular height of a triangle is 2cm greater than its base. If its area is  $35\text{cm}^2$ , use the method of trial and improvement to calculate its height, correct to 1 decimal place. Show all your calculations.
- 4) A cube has a volume of  $36\text{cm}^3$ . Use the method of trial and improvement to calculate the length of one of its sides, correct to 1 decimal place. Show all your calculations.
- 5) A cuboid has a height and length which is 3cm longer than its width. Its volume is  $150\text{cm}^3$ . Use the method of trial and improvement to calculate its width, correct to 1 decimal place. Show all your calculations.
- 6) The solution to the equation  $x^2 + 3x = 7$  lies between 1 and 2. Use the method of trial and improvement to calculate its value, correct to 1 decimal place. Show all your calculations.
- 7) Using the method of trial and improvement, solve the equation  $x^3 + 3x - 22 = 0$ , correct to one decimal place. Show all your calculations.
- 8) The solution to the equation  $x^3 + 4x = 48$  lies between 3 and 4. Use a method of trial and improvement to find the solution to  $x^3 + 4x = 48$ , correct to 1 decimal place. Show all your calculations.

## 16. Iteration

1) Starting with  $x = 4$  and using the iteration  $x_{n+1} = 4 - \frac{1}{x_n}$  find a solution of  $x = 4 - \frac{1}{x}$  correct to two decimal places.

2) Starting with  $x = 5$  and using the iteration  $x_{n+1} = 7 - \frac{9}{x_n}$  find a solution of  $x = 7 - \frac{9}{x}$  correct to two decimal places.

3) Starting with  $x = 4$  and using the iteration  $x_{n+1} = 3 + \frac{3}{x_n}$  find a solution of  $x = 3 + \frac{3}{x}$  correct to two decimal places.

4) Show that  $x = 9 - \frac{2}{x}$  can be re-arranged into the equation  $x^2 - 9x + 2 = 0$ .

Use the iterative formula  $x_{n+1} = 9 - \frac{2}{x_n}$  together with a starting value of  $x_1 = 9$  to obtain

a root of the equation  $x^2 - 9x + 2 = 0$ .

5) Show that  $x = \frac{1}{x+1}$  can be re-arranged into the equation  $x^2 + x - 1 = 0$ .

Use the iterative formula  $x_{n+1} = \frac{1}{x_n + 1}$  together with a starting value of  $x_1 = 0.5$  to

obtain a root of the equation  $x^2 + x - 1 = 0$  correct to 2 decimal places.

6) Show that  $x = \frac{12}{x+2} - 4$  can be re-arranged into the equation  $x^2 + 6x - 4 = 0$ .

Use the iterative formula  $x_{n+1} = \frac{12}{x_n + 2} - 4$  together with a starting value of  $x_1 = -6$  to

obtain a root of the equation  $x^2 + 6x - 4 = 0$  correct to 1 decimal place.

7) A sequence is given by the iteration  $x_{n+1} = \frac{21}{x_n - 4}$

a) (i) The first term,  $x_1$ , of the sequence is  $-2$ . Find the next 4 terms, correct to two decimal places.

(ii) What do you think the value of  $x$  is as  $n$  approaches infinity?

b) Show that the equation this solves is  $x^2 - 4x - 21 = 0$ .

8) A sequence is given by the iteration  $x_{n+1} = 6 + \frac{7}{x_n}$

a) (i) The first term,  $x_1$ , of the sequence is  $5$ . Find the next 3 terms.

(ii) What do you think the value of  $x$  is as  $n$  approaches infinity?

b) Show that the equation this solves is  $x^2 - 6x - 7 = 0$ .



## 18. Solving Equations 1

### Exercise 1

Solve the following equations

- |  |  |
|--|--|
| 1) $7x - 3 = 60$                                 | 2) $12x - 14 = 130$                              |
| 3) $24 - 3x = 6$                                 | 4) $7 - 2x = -1$                                 |
| 5) $\frac{x}{4} = 30$                            | 6) $\frac{x}{6} = 25$                            |
| 7) $\frac{2}{3}x = 8$                            | 8) $\frac{1}{2}x = 4$                            |
| 9) $\frac{3}{x+2} = 1$                           | 10) $\frac{2}{x-1} = 5$                          |
| 11) $2x + 3 = 3x + 2$                            | 12) $4x + 3 = 3x + 5$                            |
| 13) $4x + 5 = 3x + 3$                            | 14) $7x + 3 = 10x - 6$                           |
| 15) $5(x + 3) = 7x + 5$                          | 16) $3(2x - 1) = 5(3x - 15)$                     |
| 17) $3(x + 2) - 2(3x - 5) = 10$                  | 18) $4(2x - 3) - 3(3x - 10) = 11$                |
| 19) $\frac{x}{3} + \frac{x}{4} = 14$             | 20) $\frac{x-2}{3} + \frac{x-2}{2} = 25$         |
| 21) $\frac{2x-1}{3} - \frac{x+1}{4} = 4$         | 22) $\frac{3}{x} = \frac{4}{x+2}$                |
| 23) $\frac{2}{x-1} = \frac{3}{2x+4}$             | 24) $\frac{2x-1}{x} = \frac{3}{4}$               |
| 25) $\frac{1}{x} = \frac{3}{2x} + \frac{1}{x+1}$ | 26) $\frac{2}{3x} = \frac{1}{x} - \frac{1}{x+1}$ |

### Exercise 2

Solve the following equations

- |   |  |
|---|--|
| 1) $x^2 - 25 = 0$                               | 2) $x^2 - 81 = 0$                                |
| 3) $2x^2 - 72 = 0$                              | 4) $3x^2 - 27 = 0$                               |
| 5) $(x + 2)(x - 3) = 0$                         | 6) $(x - 6)(x + 5) = 0$                          |
| 7) $(3x + 4)(2x - 1) = 0$                       | 8) $x(3x - 2) = 0$                               |
| 9) $x(4x + 3) = 0$                              | 10) $2x(x - 4) = 0$                              |
| 11) $2x^2 + 3x = 0$                             | 12) $6x^2 - 4x = 0$                              |
| 13) $4x^2 - x = 0$                              | 14) $x^2 - x - 6 = 0$                            |
| 15) $2x^2 - x - 3 = 0$                          | 16) $6x^2 + 4x - 2 = 0$                          |
| 17) $x^2 - 3x - 10 = 0$                         | 18) $3x^2 + 6x = 24$                             |
| 19) $8x^2 = 2x + 15$                            | 20) $4x^2 - 25 = 0$                              |
| 21) $\frac{4}{x} + \frac{1}{x+1} = \frac{6}{x}$ | 22) $\frac{1}{x} = \frac{3}{2x} + \frac{1}{x+1}$ |



## 19. Solving Equations 2

### Exercise 1

Solve the following equations correct to 2 decimal places each time.

- |  |  |
|--|--|
| 1) $x^2 + 4x - 6 = 0$                    | 2) $x^2 - 3x + 1 = 0$                    |
| 3) $x^2 - 4x - 3 = 0$                    | 4) $x^2 + 6x - 4 = 0$                    |
| 5) $x^2 + x - 1 = 0$                     | 6) $4x^2 - 7x - 4 = 0$                   |
| 7) $6x^2 + 3x - 1 = 0$                   | 8) $10x^2 + 2x - 7 = 0$                  |
| 9) $4x^2 + 5x - 10 = 0$                  | 10) $3x^2 - 4x - 2 = 0$                  |
| 11) $3x^2 + 4x = 5$                      | 12) $2x^2 + 5x = 1$                      |
| 13) $4x^2 + 3 = 7x$                      | 14) $3x(2x + 1) = 1$                     |
| 15) $\frac{1}{x} + \frac{3}{x+2} = 7$    | 16) $\frac{1}{x-3} - \frac{1}{2x} = 4$   |
| 17) $\frac{5}{x} + \frac{1}{x+1} = 7$    | 18) $\frac{7}{2x} + \frac{2}{x-1} = 4$   |
| 19) $x^2 + 7x + 2 = 2x + 4$              | 20) $x^2 + 13x = 4x + 5$                 |
| 21) $5x^2 - 3x = x^2 + 4$                | 22) $4x^2 - 3x = 2x^2 + 7$               |
| 23) $x(x + 4) = 3x^2 + 2x - 6$           | 24) $4(x + 2) = 3x(x + 1)$               |
| 25) $\frac{4}{x} = \frac{2x - 3}{x - 1}$ | 26) $\frac{2x + 3}{x - 1} = \frac{x}{4}$ |

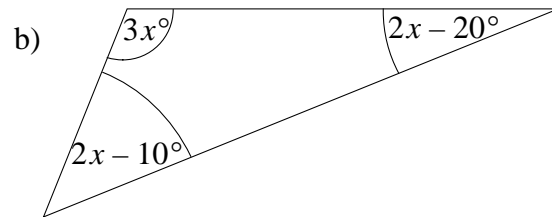
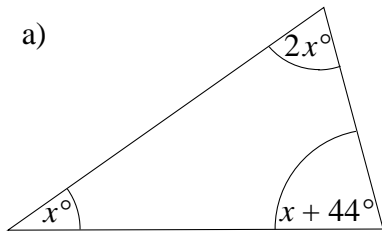
### Exercise 2

In each of the following calculate the value of  $a$  and the corresponding value of  $k$ .

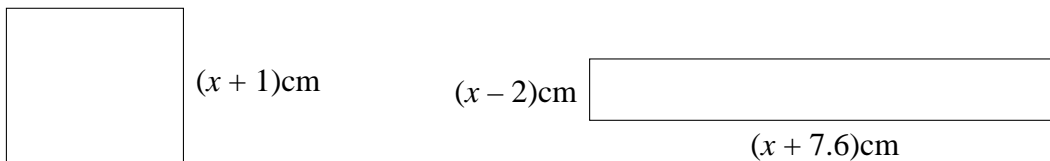
- |                                   |                                    |
|-----------------------------------|------------------------------------|
| 1) $4x^2 - 20x + k = (2x - a)^2$  | 2) $(3x + a)^2 = 9x^2 + 12x + k$   |
| 3) $(5x - a)^2 = 25x^2 - 30x + k$ | 4) $x^2 - 14x + k = (x - a)^2$     |
| 5) $(3x + a)^2 = 9x(x + 1) + k$   | 6) $16x(x + 1) + k = (4x + a)^2$   |
| 7) $9x^2 - 30x + k = (3x - a)^2$  | 8) $(4x - a)^2 = 16x(x - 4) + k$   |
| 9) $(2x + a)^2 = 4x^2 + 24x + k$  | 10) $36x^2 + 48x + k = (6x + a)^2$ |

## 20. Problems Involving Equations

1. In each of the triangles below, calculate the value of  $x$  and hence the sizes of the angles of the triangles.



2. It takes an aeroplane  $6\frac{1}{2}$  hours to travel from London to the USA, a distance of 3,500 miles.
- What was the average speed?
  - If the same aeroplane travels from London to Spain, a distance of  $x$  miles, write down in terms of  $x$  the time taken.
  - If the same aeroplane travels from London to Italy in  $y$  minutes, write down an expression in terms of  $y$  for the distance travelled.
3. The dimensions of a square and a rectangle are given in the two diagrams. If their areas are equal,
- calculate the value of  $x$
  - calculate their areas.



4. Three consecutive numbers add up to 156.
- If the middle number is  $x$ , what are the values of the other two numbers, in terms of  $x$ .
  - Write down an equation in terms of  $x$  and solve it to find  $x$ .
5. A wine merchant has  $x$  bottles of wine in her shop and  $y$  bottles in her cellar. She transfers a quarter of the bottles from the cellar to the shop.
- How many bottles does she now have (i) in the shop (ii) in the cellar?  
She now finds that she has twice as many bottles in the cellar as she has in the shop.
  - Write down an equation linking  $x$  and  $y$  and simplify it.
  - If she originally had 2,000 bottles in the shop, how many has she altogether?
6. For the annual village fete, the vicar orders 250 bottles of drinks. He orders  $x$  bottles of lemonade and the remainder of cola.  
Bottles of lemonade cost 35p each and bottles of cola cost 38p each. He spends £91.70 altogether.
- Write down in terms of  $x$  the number of bottles of cola he bought.
  - Write down an equation for the total cost of the bottles and from it calculate the value of  $x$ .
  - How many bottles of cola did he buy?
7. Three numbers are added together. The second number is 6 more than the first number and the third number is 15 less than the first number.
- If the second number is  $x$ , write down in terms of  $x$ , the value of each of the other two numbers.
  - The three numbers are added together. Write down a simplified expression for the total of the three numbers.
  - If the sum of the numbers is 93, what are the three numbers?

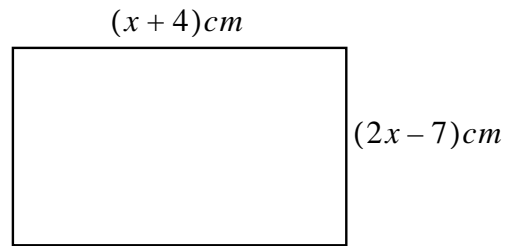
## 21. Simultaneous Equations 1

- |                                    |                                     |                                      |                                     |
|------------------------------------|-------------------------------------|--------------------------------------|-------------------------------------|
| 1) $2x + 2y = 10$<br>$x + 2y = 6$  | 2) $3x + y = 18$<br>$2x + y = 13$   | 3) $4x + 2y = 2$<br>$2x + 2y = 0$    | 4) $5x + 3y = 18$<br>$5x + y = 16$  |
| 5) $x + y = 1$<br>$x - y = 5$      | 6) $3x + 4y = 29$<br>$x - 4y = -17$ | 7) $3x - 2y = 10$<br>$-3x + y = -11$ | 8) $3x + 4y = 18$<br>$3x - 4y = -6$ |
| 9) $4x + 3y = 11$<br>$2x + y = 7$  | 10) $5x + 2y = 33$<br>$2x + y = 14$ | 11) $6x + 2y = 10$<br>$4x + y = 7$   | 12) $3x - 2y = 13$<br>$x - y = 5$   |
| 13) $2x + 3y = 28$<br>$3x - y = 9$ | 14) $2x + 3y = 15$<br>$5x - y = 46$ | 15) $4x + 3y = 13$<br>$6x - 2y = 13$ | 16) $5x + 3y = 14$<br>$2x + 2y = 4$ |

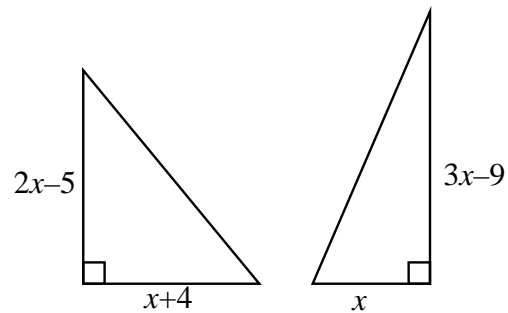
- 17) A family of 2 adults and 2 children go to the cinema. Their tickets cost a total of £14.00. Another family of 1 adult and 4 children go to the same cinema and their bill is £13.60.
- Letting  $x$  represent the cost of an adults ticket and  $y$  the cost of a childs ticket, write down two equations connecting  $x$  and  $y$
  - Solve for  $x$  and  $y$ .
- 18) The sum of two numbers is 39 and their difference is 9.
- Letting  $x$  and  $y$  be the two numbers, write down two equations connecting  $x$  and  $y$ .
  - Solve the equations.
- 19) A rectangle has a perimeter of 42cm. Another rectangle has a length double that of the first and a width one third of that of the first. The perimeter of the second is 57cm. Letting  $x$  and  $y$  represent the dimensions of the first rectangle, write down two equations containing  $x$  and  $y$ . Solve the equations and write down the dimensions of the second rectangle.
- 20) 4 oranges and 3 apples weigh 720 grams. 3 oranges and 4 apples weigh 750 grams. Let  $x$  and  $y$  represent their weights. Write down two equations containing  $x$  and  $y$ . Calculate the weights of each piece of fruit.
- 21) Three mugs and two plates cost £7.20, but four mugs and one plate cost £7.90. Let  $x$  represent the cost of a mug and  $y$  the cost of a plate. Write down two equations involving  $x$  and  $y$ . Solve these equations and calculate the cost of seven mugs and six plates.
- 22) Sandra withdrew £400 from the bank. She was given £20 and £10 notes, a total of 23 altogether. Let  $x$  represent the number of £20 notes and  $y$  the number of £10 notes. Write down two equations and solve them.
- 23) A quiz game has two types of question, hard ( $h$ ) and easy ( $e$ ). Team A answers 7 hard questions and 13 easy questions. Team B answers 13 hard questions and 3 easy questions. If they both score 74 points, find how many points were given for each of the two types of question.
- 24) A man stays at a hotel. He has bed and breakfast ( $b$ ) for three nights and two dinners ( $d$ ). A second man has four nights bed and breakfast and three dinners. If the first man's bill is £90 and the second man's bill is £124, calculate the cost of a dinner.
- 25) Four large buckets and two small buckets hold 58 litres. Three large buckets and five small buckets hold 68 litres. How much does each bucket hold?
- 26) Caroline buys three first class stamps and five second class stamps for £1.94. Jeremy buys five first class stamps and three second class stamps for £2.06. Calculate the cost of each type of stamp.

## 22. Problems Involving Quadratic Equations

1. A rectangle has a length of  $(x + 4)$  centimetres and a width of  $(2x - 7)$  centimetres.
- a) If the perimeter is 36cm, what is the value of  $x$ ?
- b) If the area of a similar rectangle is  $63\text{ cm}^2$ , show that  $2x^2 + x - 91 = 0$  and calculate the value of  $x$ .



2. The areas of the two triangles on the right are equal.
- a) Write down an equation in  $x$  and simplify it.
- b) Solve this equation and calculate the area of one of the triangles.



3. Bill enters a 30 kilometre race. He runs the first 20km at a speed of  $x$  kilometres per hour and the last 10km at  $(x - 5)$  kph. His total time for the race was 4 hours.
- a) Write down an equation in terms of  $x$  and solve it.
- b) What are his speeds for the two parts of the run?
4. A piece of wire is cut into 2 parts. The first part is bent into the shape of a square. The second part is bent into the shape of a rectangle with one side 4cm long and the other side twice the length of the square's side. Let  $x$  represent one side of the square.
- a) Write down two expressions in  $x$  for the areas of the two shapes.
- b) If the sum of the two areas is  $105\text{ cm}^2$ , show that  $x^2 + 8x - 105 = 0$ .
- c) Calculate the length of the original wire.
5. A small rectangular lawn is twice as long as it is wide. It has a path around it which is 2 metres wide. The area of the path is twice the area of the lawn
- a) If the small side of the lawn is  $x$  metres, write down the dimensions of the outside edge of the path.
- b) By writing down the area of the lawn in terms of  $x$  and using the answer to part a), form an equation in  $x$ .
- c) Simplify this equation so that it can be written as  $4(x^2 - 3x - 4) = 0$  and solve it.
- d) Write down the dimensions of the lawn.
6. Bill and Dan take part in a fun run. Bill's average speed is  $(x + 4)$  kph and Dan's is  $(3x)$  kph. Bill completes his run in  $(x - 1)$  hours and Dan in  $(x - 2)$  hours.
- a) Write down two expressions representing the distance travelled by both runners.
- b) Combine these expressions to find the value of  $x$ .
- c) What was Bill's speed?

## 23. Straight Line Graphs

### Exercise 1

In each of the following questions say what the gradient of the line is.

- 1)  $y = x + 2$       2)  $y = 3x + 3$       3)  $y = -5x - 2$       4)  $y = 3 - 4x$   
 5)  $2y = 4x + 3$       6)  $3y = 6x - 5$       7)  $3y = -5x + 2$       8)  $4y = 6 - x$   
 9)  $y - 2x + 3 = 0$       10)  $2x - y - 3 = 0$       11)  $y + x + 3 = 0$       12)  $2y + 2x - 2 = 0$   
 13)  $2y + 2x = 1$       14)  $x + y = 3$       15)  $4x - 3y = 4$       16)  $x - 3y = 2$

### Exercise 2

- 1) What is the equation of the line parallel to  $y = x$  which goes through the point (3,0)?
- 2) What is the equation of the line parallel to  $y = 2x$  which goes through the point (5,0)?
- 3) What is the equation of the line parallel to  $y = 3x + 2$  which goes through the point (0,0)?
- 4) What is the equation of the line parallel to  $y = -x - 6$  which goes through the point (0,2)?
- 5) What is the equation of the line parallel to  $y = -3x + 2$  which goes through the point (1,1)?
- 6) What is the equation of the line parallel to  $y = 4x - 7$  which goes through the point (4,2)?

### Exercise 3

- 1) The table below shows the relationship between  $x$  and  $y$ .

$x$	10	20	30	40	50
$y$	14	34	54	74	94

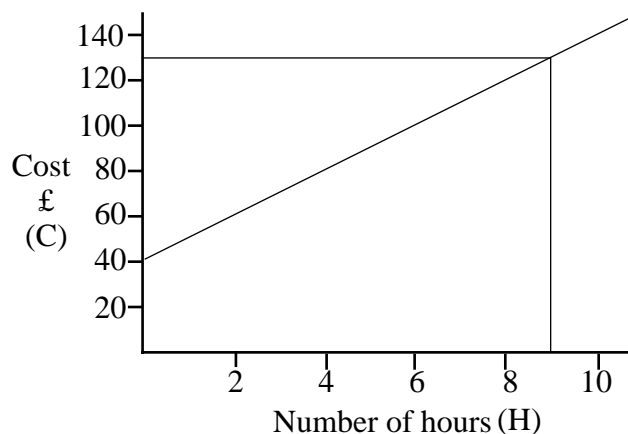
- a) On graph paper plot the values of  $x$  and  $y$  and show that this graph is of the form  $y = mx + c$
  - b) What are the values of  $m$  and  $c$ ?
  - c) What is the value of  $y$  when  $x = 65$ ?
  - d) What is the value of  $x$  when  $y = 234$ ?
- 2) The table below shows the relationship between two sets of values  $x$  and  $y$ .

$x$	2	4	6	8	10
$y$	30	40	50	60	70

- a) On graph paper plot the values of  $x$  and  $y$  and show that this graph is of the form  $y = mx + c$
- b) What are the values of  $m$  and  $c$ ?
- c) What is the value of  $y$  when  $x = 15$ ?
- d) What is the value of  $x$  when  $y = 105$ ?

- 3) A coach company charges for the hire of their coaches according to the graph shown.

- a) Write down the equation to the line
- b) What is the cost of hiring a coach for 5 hours?
- c) What is the cost of hiring a coach for  $6\frac{1}{2}$  hours?
- d) For how many hours is a coach hired for if it costs £100?



## 24. Inequalities

### Exercise 1

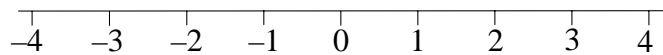
In each of the following inequalities, solve them in the form  $x > n$ ,  $x < n$ ,  $x \geq n$  or  $x \leq n$  where  $n$  is a number.

- |                                |                                |                                |
|--------------------------------|--------------------------------|--------------------------------|
| 1) $x + 1 > 7$                 | 2) $x + 4 > 6$                 | 3) $x + 12 > 20$               |
| 4) $x + 6 > 3$                 | 5) $x + 5 > 2$                 | 6) $x + 7 > 9$                 |
| 7) $2x > 12$                   | 8) $3x > 12$                   | 9) $4x > 20$                   |
| 10) $x + 3 < 5 - x$            | 11) $x + 4 \leq 10 - x$        | 12) $x + 3 \leq 13 - x$        |
| 13) $3x - 9 \geq x + 7$        | 14) $5x - 6 < 2x + 3$          | 15) $6x - 8 < 4x + 2$          |
| 16) $2(x - 3) > 3(2 - x)$      | 17) $3(x + 1) \leq 2(x + 7)$   | 18) $5(x + 3) \leq 2(x - 4)$   |
| 19) $5(2x - 4) \leq 3(3x - 7)$ | 20) $3(2x - 7) \leq 4(2x + 8)$ | 21) $3(2x - 6) \geq 2(4x - 7)$ |

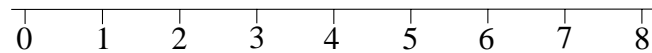
### Exercise 2

In questions 1 to 3, copy the number line into your book. Mark on it the integer values of  $x$  which satisfy the inequality.

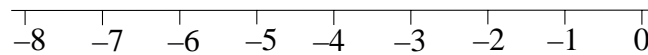
1.  $2x + 3 > 3x + 4$



2.  $3x + 7 > 4x + 2$

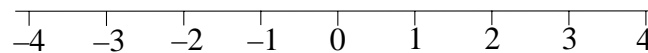


3.  $4x + 7 < 2x - 6$

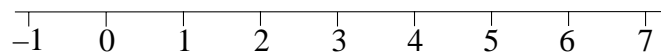


In questions 4 and 5, copy the number line into your book. Mark on the number line the range of values which satisfy the inequality

4.  $4x + 3 > 2x - 4$



5.  $5x - 6 < 2x + 4$



6. If  $3x + 4 > x + 7$ , what is the least whole number that satisfies this inequality?
7. If  $3x + 4 < x + 7$ , what is the greatest whole number that satisfies this inequality?
8. A social club want to hire a bus to take them out for the day. Company A charge £20 per hour. Company B charge £70 plus £10 per hour. Let  $x$  be the number of hours they want to hire the bus for.
  - a) Write down the inequality satisfied by  $x$  where the cost of the number of hours charged by company A is less than the cost charged by company B.
  - b) Solve the inequality and explain what the solution tells you.

## 25. Linear Inequalities

1. Plot the graphs of  $x = 6$ ,  $y = 10 - x$  and  $y = \frac{x+8}{2}$  for values of  $x$  from 0 to 6 and values of  $y$  from 0 to 10. Clearly show the area satisfied by the inequalities  $x \leq 6$ ,  $y \geq 10 - x$  and  $y \leq \frac{x+8}{2}$ . Write down the integer points within this area.
2. Plot the graphs of  $y = 6$ ,  $y = 6 - 2x$  and  $y = 4x - 2$  for values of  $x$  from 0 to 4 and values of  $y$  from  $-3$  to 7. Clearly show the area satisfied by the inequalities  $y \leq 6$ ,  $y \geq 6 - 2x$  and  $y \geq 4x - 2$ . Write down the integer points within this area.
3. For values of 0 to 5 on the  $x$  axis and 0 to 12 on the  $y$  axis, plot the graphs of  $y + \frac{x}{3} = 4$ ,  $y = \frac{1}{2}x + 3$  and  $y = 12 - 3x$ . On your diagram clearly indicate the area satisfied by the inequalities  $y + \frac{x}{3} \geq 4$ ,  $y \leq \frac{1}{2}x + 3$  and  $y \leq 12 - 3x$ . List the integer points lying within this area.
4. Plot the graphs of  $y = x$ ,  $y = 12 - 3x$  and  $y = 7 - \frac{1}{2}x$  for values of  $x$  from 0 to 6 and values of  $y$  from 0 to 12. Clearly indicate the area satisfied by the inequalities  $y > x$ ,  $y > 12 - 3x$  and  $y < 7 - \frac{1}{2}x$ . Write down the integer points lying within this area.
5. Plot the graphs representing the equations  $x + y = 10$ ,  $y + \frac{1}{2}x = 5$ ,  $y = \frac{1}{2}x + 4$  and  $y = 2x - 3$  for values of  $x$  from 0 to 6 and values of  $y$  from  $-3$  to 10. On your diagram shade in the region representing the inequalities  $x + y < 10$ ,  $y + \frac{1}{2}x > 5$ ,  $y \leq \frac{1}{2}x + 4$  and  $y > 2x - 3$ . Write down the integer points lying within this region.
6. On graph paper draw lines and shade in the area representing the inequalities  $y \leq 8 - 2x$ ,  $y \geq 6 - 3x$ ,  $y \leq \frac{1}{2}x - 1$  and  $y \geq x - 4$  for values of  $x$  from 0 to 6 and  $y$  from  $-4$  to 8. Write down the integer points lying within this region.
7. Plot lines and shade in the area representing the inequalities  $y + \frac{1}{2}x < 1$ ,  $y \geq x - 3$ ,  $y < 2x - 2$  and  $y > -\frac{1}{3}(x + 5)$  for values of  $x$  from  $-5$  to 4 and  $y$  from  $-3$  to 2. Write down the integer points lying within this region.
8. Plot lines and shade in the area representing the inequalities  $y + 2x < 8$ ,  $y < x + 3$ ,  $y + \frac{1}{2}x > 2$  and  $y > 2x - 2$  for values of  $x$  from  $-1$  to 5 and values of  $y$  from  $-2$  to 8. Write down the integer points lying within this area.
9. Plot lines and shade in the area representing the inequalities  $y > x + 1$ ,  $y + 3x < 7$ ,  $y < 2x + 6$  and  $y > 1 - x$  for values of  $x$  from  $-3$  to 4 and values of  $y$  from  $-3$  to 8. Write down the integer points lying within this area.

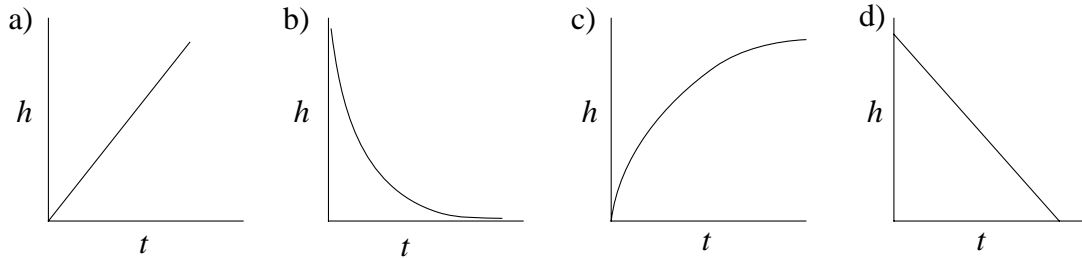
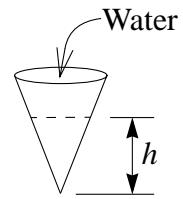
## 26. Linear Programming

- 1) A van can carry a maximum load of 400 kg. It carries boxes weighing 20 kg and 40 kg. It carries at least 7 boxes weighing 40 kg. The number of boxes weighing 40 kg is not more than twice the number of 20 kg boxes.  
Let  $x$  represent the number of 20 kg boxes and  $y$  the number of 40 kg boxes.
- Write down three inequalities involving  $x$  and  $y$ .
  - Illustrate the three inequalities by a suitable diagram on graph paper. Let 2 cm represent 1 box on both axes.
  - From the diagram determine the least weight the van carries.
  - What combinations give the greatest weight?
- 2) Orange is produced by mixing together red and yellow in certain ratios. David wants to make up to 10 litres of orange paint. He mixes together 200 ml tins of red and yellow. To be classified as orange there must not be more than twice as much of one colour than the other. Let  $x$  represent the number of red tins and  $y$  the number of yellow tins.
- Write down 3 inequalities in  $x$  and  $y$ .
  - On graph paper, illustrate these inequalities using a scale of 2 cm to represent 10 tins on each axis, showing clearly the area representing the orange mix.
  - How much of each colour would he use to make 10 litres if he wanted;
    - the reddest possible shade of orange.
    - the yellowest possible shade of orange.
  - What is the maximum amount of orange paint he can make with 14 tins of red?
- 3) A factory produces curtains for large and small windows. Each large curtain requires 10 m<sup>2</sup> of fabric and each small curtain requires 5 m<sup>2</sup> of fabric. There is a total of 500 m<sup>2</sup> of fabric available each day.  
For each of the large curtains the factory makes a profit of £5 and for each small curtain it makes a profit of £8. To cover costs the factory needs to make a profit of at least £400 each day. Due to the type of demand for the curtains, they never make more than twice as many small curtains as large ones.  
Let  $x$  represent the number of large curtains and  $y$  the number of small curtains.
- Write down three inequalities involving  $x$  and  $y$ .
  - Represent these inequalities on a graph and clearly indicate the area which satisfies these inequalities. Use a scale of 2 cm to represent 10 curtains on each axis.
  - From the diagram find the values of  $x$  and  $y$  which will satisfy all the three conditions and give the greatest profit.
- 4) The dimensions of a rectangle are such that its perimeter is greater than 20 metres and less than 30 metres. One side must be greater than the other. The larger side must be less than twice the size of the smaller side.  
Let  $x$  represent the length of the smaller side and  $y$  the length of the larger one.
- Write down four inequalities involving  $x$  and  $y$ .
  - On graph paper, illustrate these inequalities using a scale of 2 cm to represent 2 metres on each axis, clearly showing the area containing the solution.
  - What whole number dimensions will satisfy these three inequalities?

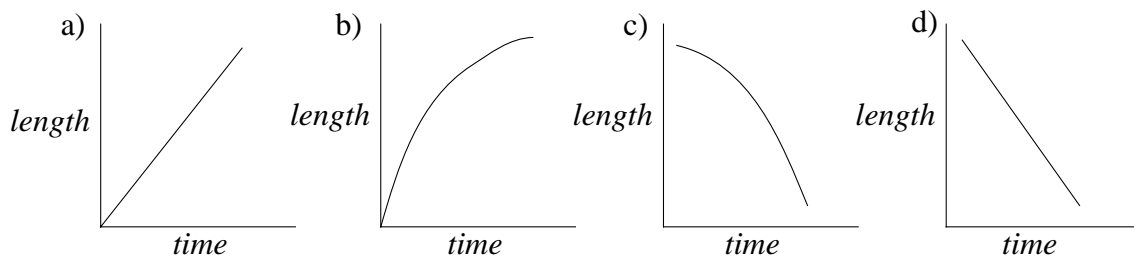


### 27. Recognising Graphs.

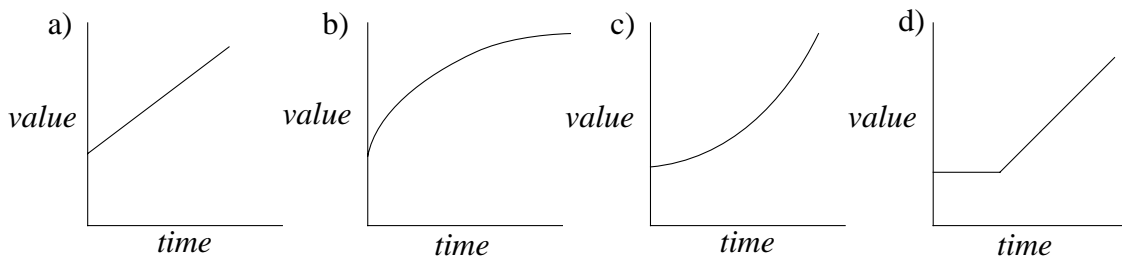
1. Water runs into a conical container. The height ( $h$ ) of the water is plotted against the time ( $t$ ) it takes for the water to flow into the container. Which of the following sketches represents this?



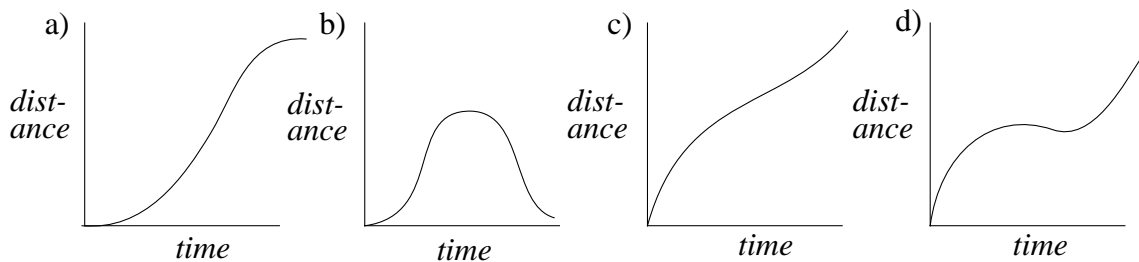
2. A weight is suspended from the bottom end of a piece of wire. The top end is fixed. The weight makes the wire extend at a constant rate. Which of the following diagrams shows this?



3. David buys an antique table. He estimates that each year its value will increase by 20% of its value at the beginning of that year. Which of the following diagrams represents this?



4. A car sets out from town A and drives to town B. The car is slowed down by traffic at the beginning and the end of the journey, but speeds up in the middle section. Which of the following diagrams shows this?



## 28. Graphs 1

1. a) Complete this table for values of  $y = x^2 + 2x - 3$ .

$x =$	-4	-3	-2	-1	0	1	2
$y = x^2 + 2x - 3$							

- b) Draw the graph of  $y = x^2 + 2x - 3$  using a scale of 2cm to represent 1 unit on the  $x$  axis and 2cm to represent 1 unit on the  $y$  axis.
- c) Using the same axis, draw the graph of  $y = x + 2$ .
- d) Write down the two values of  $x$  where the two graphs cross.
- e) Write down a simplified equation which satisfies these two values of  $x$ .
2. a) Complete this table for the values of  $y = x^3 - 2x + 2$ .

$x =$	-2.5	-2	-1	0	1	2	2.5
$y = x^3 - 2x + 2$	-8.625		3			6	

- b) Plot the graph of  $y = x^3 - 2x + 2$  using a scale of 2cm to represent 1 unit on the  $x$  axis and 2cm to represent 2 units on the  $y$  axis.
- c) On the same axes, draw the graph of  $y = 3x + 2$ .
- d) Show on the graph that the equation  $x^3 - 5x = 0$  has three solutions. From the graphs give approximate values of these solutions.
3. a) Complete the table of values for  $y = 2x^2 - 4x - 3$ .

$x =$	-2	-1	0	1	2	3	4
$y = 2x^2 - 4x - 3$							

- b) Plot the graph of  $y = 2x^2 - 4x - 3$  using a scale of 2cm to represent 1 unit on the  $x$  axis and 2cm to represent 2 units on the  $y$  axis.
- c) From the graph write down the solution to the equation  $2x^2 - 4x - 3 = 0$ .
- d) On the graph draw in the line  $y = 4$  and from the graph write down the approximate solution to the equation  $2x^2 - 4x - 7 = 0$ .
4. a) Complete the table of values for  $y = (x + 3)(x - 1)$

$x =$	-4	-3	-2	-1	0	1	2
$x+3$							
$x-1$							
$y = (x + 3)(x - 1)$							

- b) Using a scale of 2cm to represent 1 unit on both axes, draw the graph of  $y = (x + 3)(x - 1)$ .
- c) On the same axes, draw the graph of  $y = x + 1$ .
- d) From your graph, estimate the  $x$  co-ordinates of the points of intersection of the two graphs and write down the quadratic equation which these values of  $x$  satisfy.

## 29. Graphs 2

1. a) Complete the table of values of  $y = 2x + \frac{8}{x}$  for the values of  $x$  from 0.5 to 6.

$x$	0.5	1.0	1.5	2.0	2.5	3	4	6	7
$y = 2x + \frac{8}{x}$									

- b) Using a scale of 2cm to represent 1 unit on the  $x$  axis and 1cm to represent 1 unit on the  $y$  axis plot the graph of  $y = 2x + \frac{8}{x}$ .
- c) Using the same axes draw the lines representing  $y = 14$  and  $y = 12 - \frac{x}{2}$ .
- d) By considering the points of intersection of two graphs write down the approximate solutions to the equation  $2x + \frac{8}{x} - 14 = 0$ .
- e) Show that the intersection of the graphs  $y = 2x + \frac{8}{x}$  and  $y = 12 - \frac{x}{2}$  gives a solution to the equation  $5x^2 - 24x + 16 = 0$ . What are the approximate solutions to this equation?
- f) What is the gradient of the curve  $y = 2x + \frac{8}{x}$  when  $x = 4$ ?
2. a) Complete the table of values of  $y = 4 + 3x - x^2$  for values of  $x$  from  $-4$  to  $+4$ .

$x =$	-4	-3	-2	-1	0	1	2	3	4	5
$y = 4 + 3x - x^2$										

- b) Draw the graph of  $y = 4 + 3x - x^2$  using a scale of 2cm to represent 1 unit on the  $x$  axis and 2cm to represent 4 units on the  $y$  axis.
- c) On the same axes draw the line  $y = 3$  and write down the approximate co-ordinates of the point of intersection of the two graphs.
- d) Show that the  $x$  co-ordinates at this point are an approximate solution to the equation  $3x - x^2 + 1 = 0$ .
- e) What is the solution to the equation  $4 + 3x - x^2 = 0$ ?
- f) By drawing a straight line, find an approximate solution to the equation  $8 + 3x - x^2 = 0$ .
3. Draw the graphs of  $y = (x + 3)(3 - 2x)$  and  $y = \frac{3x + 5}{2}$  for values of  $x$  from  $-4$  to  $+2$  using a scale of 2cm to 1 unit on the  $x$  axis and 1 cm to 2 unit on the  $y$  axis. From your graph estimate the solutions to the equations;
- a)  $13 - 9x - 4x^2 = 0$  and
- b)  $3 - 3x - 2x^2 = 0$

### 30. Graphs 3

1. The table below shows values of  $y = x^2 + c$ . What is the value of  $c$ ?

$x$	1	2	3	4	5
$y$	2.5	5.5	10.5	17.5	26.5

2. The table below shows values of  $y$  which are approximately equal to  $ax^2 + b$ , where  $a$  and  $b$  are constants.

$x$	1	2	4	6	8
$y$	53	62	98	160	240

- Plot the values of  $y$  against  $x$ , using a scale of 2cm to represent 1 unit on the  $x$  axis and 2cm to represent 20 units on the  $y$  axis.
  - From your graph, determine the approximate values of  $a$  and  $b$ .
  - What is the approximate value of  $y$  when  $x = 7$ ?
3. The table shows the approximate values of  $y$  which satisfy the equation  $y = pq^x$ , where  $p$  and  $q$  are constants.

$x$	0	0.5	1	1.5	2	2.5	3
$y$	3	4.24	6	8.5	12	17	24

- Plot the values of  $y$  against  $x$ , using a scale of 4cm to represent 1 unit on the  $x$  axis and 2cm to represent 2 units on the  $y$  axis.
  - Use your graph to help you estimate the values of  $p$  and  $q$ .
  - What is the approximate value of  $y$  when  $x = 2.25$ ?
4. The table below shows the approximate values of  $y$  which satisfy the equation  $y = a \sin x + b$  where  $a$  and  $b$  are constants.

$x^\circ$	0	30	60	90	120	150	180
$y$	-0.5	0.25	0.8	1.0	0.8	0.25	-0.5

- Using a scale of 2cm to represent  $30^\circ$  on the  $x$  axis and 10cm to represent 1 unit on the  $y$  axis, plot the graph of  $y = a \sin x + b$
  - From your graph estimate the values of  $a$  and  $b$ .
  - What is the approximate value of  $y$  when  $x = 45^\circ$ ?
5. The table below shows the approximate values of  $y$  which satisfy the equation  $y = a^x + b$  where  $a$  and  $b$  are constants.

$x$	0	0.5	1.0	1.5	2.0	2.5
$y$	3.0	3.73	5.0	7.2	11	17.6

- Draw the graph of  $y = a^x + b$ . Allow 4cm to represent 1 unit in the  $x$  axis and 2cm to represent 2 units on the  $y$  axis.
- From your graph estimate the values of  $a$  and  $b$ .
- What is the approximate value of  $y$  when  $x = 1.3$ ?

### 31. Growth and Decay

1. The relationship between  $x$  and  $y$  is given by the equation  $y = 1.5^{-x}$ .

(a) Complete the table, giving  $y$  correct to 3 decimal places where necessary.

$x$	0	1	2	3	4	5	6
$y$		0.667			0.198		

(b) Draw the graph of  $y = 1.5^{-x}$ , allowing 2cm to represent 1 unit on the  $x$  axis and 2cm to represent 0.1 on the  $y$  axis.

(c) From your graph, estimate the following, showing clearly where your readings are taken.

(i) the value of  $x$  when  $y = 0.7$ .

(ii) the value of  $y$  when  $x = 3.5$ .

2. The population of a country grows over a period of 7 years according to the equation

$P = P_0 \times 1.1^t$  where  $t$  is the time in years,  $P$  is the population after time  $t$  and  $P_0$  is the initial population.

a) If  $P_0 = 10$  million, complete the table below giving your values correct to 2 decimal places where necessary.

$t$	0	1	2	3	4	5	6	7
$P$ (million)	10				14.64			19.49

b) Plot  $P$  against  $t$ . Allow 2cm to represent 1 year on the horizontal axis and 2cm to represent 1 million on the vertical axis (Begin the vertical axis at 8 million).

c) From your diagram estimate the population after  $5\frac{1}{2}$  years.

d) How long will the population take to reach 15 million?

3. The percentage of the nuclei remaining in a sample of radioactive material after time  $t$  is given by the formula  $P = 100 \times a^{-t}$ , where  $P$  is the percentage of the nuclei remaining after  $t$  days and  $a$  is a constant.

a) Copy and complete the table below for  $a = 3$ .

$t$	0	0.5	1	1.5	2	2.5	3	3.5	4
$P$	100			19.3			3.7		

b) Draw a graph showing  $P$  vertically and  $t$  horizontally. Use a scale of 4cm to represent 1 day on the horizontal axis and 2cm to represent 10% on the vertical axis.

c) From the graph, estimate the following, showing clearly where your readings are taken.

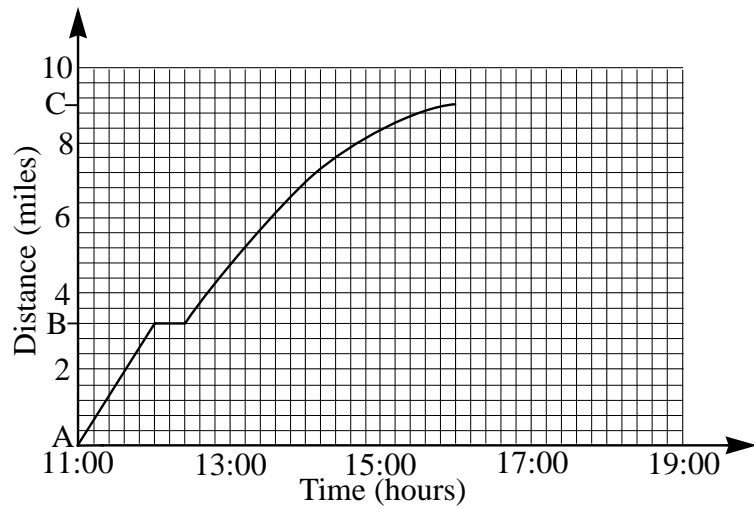
(i) The half life of the material (ie when 50% of the nuclei remain) correct to the nearest hour.

(ii) The percentage of the sample remaining after 2.25 days.

(iii) The time at which three times as much remains as has decayed.

### 32. Distance - Time Diagrams

- The graph shows a journey undertaken by a group of walkers. From the graph determine
  - their average speed between points A and B.
  - their average speed between points B and C.
  - their approximate speed at 2pm.
  - At C they rest for half an hour and then return to A at a constant speed. If they arrive home at 8.00pm, what is their average speed?



- An object is projected vertically upwards so that its height above the ground  $h$  in time  $t$  is given in the following table.

Time $t$ seconds	0	0.5	1	1.5	2	2.5	3	3.5	4
Height $h$ metres	0	7	12	15	16	15	12	7	0

Draw a graph to show this information using a scale of 4cm to represent 1 second on the horizontal axis and 2cm to represent 2 metres on the vertical axis. From your graph find

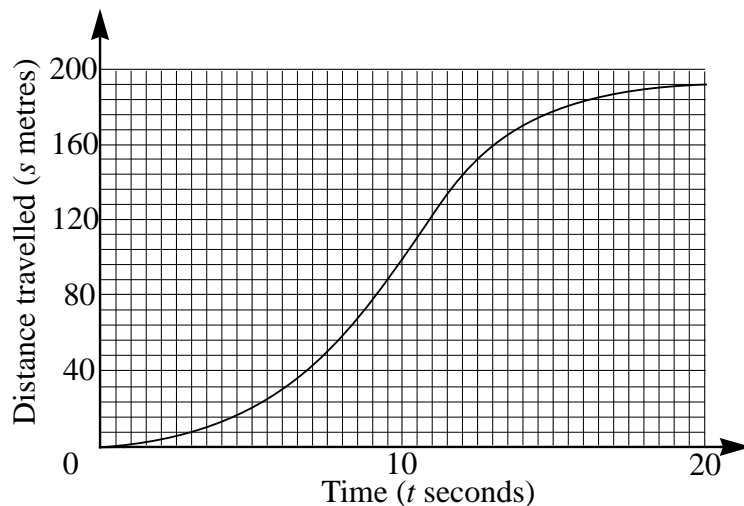
- the time, to the nearest 0.1 second, it takes for the object to reach 10 metres.
- the velocity of the object when  $t = 1.7$ secs.

- The curve shows the distance travelled ( $s$ ) by a car in time ( $t$ ).

- Find its approximate speed when  $t = 14$  seconds.

Explain what is happening to the car between

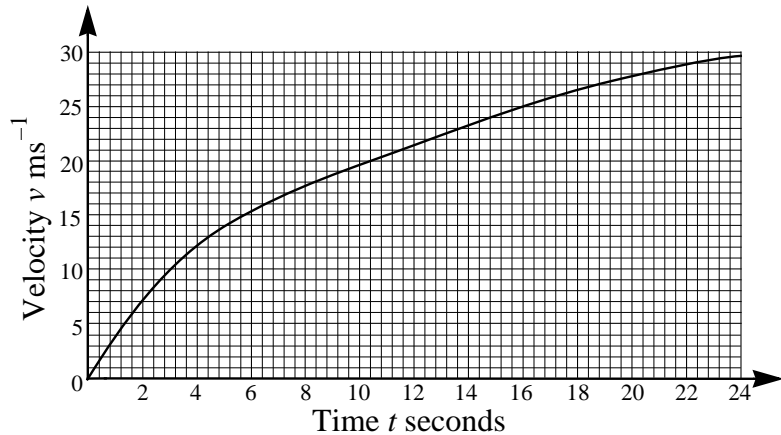
- $t = 0$  and  $t = 8$  secs
- $t = 8$  secs and  $t = 12$  secs
- $t = 12$  secs and  $t = 20$  secs



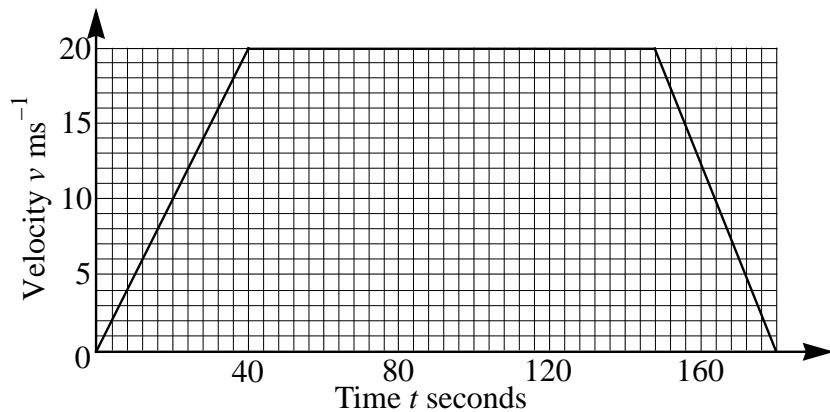
- An object is projected vertically upwards. Its height  $h$  above the ground after time  $t$  is given by the formula  $h = 30t - 6t^2$  where  $h$  is measured in metres and  $t$  is in seconds. Draw a graph to show this relationship for values of  $t$  from 0 to 5 seconds. From your graph find
  - the height when  $t = 1.4$  seconds.
  - the approximate speed of the object when  $t = 2$  seconds.
  - the distance travelled in the fourth second.
  - the maximum height gained by the object.

### 33. Velocity-Time Graphs

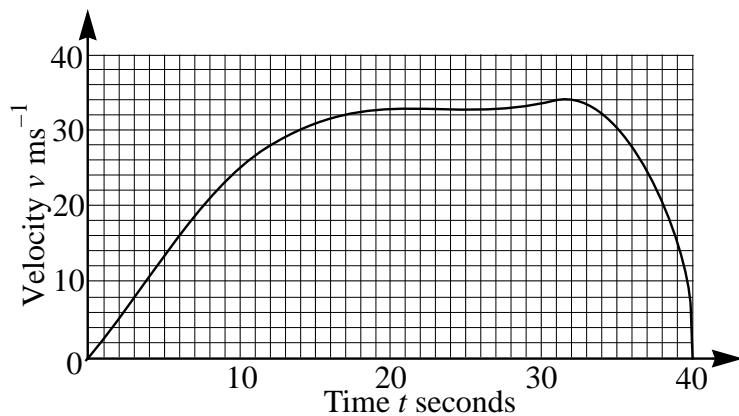
1. The velocity of a vehicle after time  $t$  seconds is given by the graph on the right.
  - a) Find the area underneath the graph between  $0 < t < 20$  seconds.
  - b) What is the approximate distance travelled by the vehicle in this time?
  - c) What is the acceleration of the vehicle when the velocity is  $10\text{ms}^{-1}$ ?



2. The diagram below shows a velocity-time graph for the journey a car makes. Use it to calculate
  - a) the total distance travelled in 180 seconds
  - b) the average acceleration in the first 40 seconds.



3. The diagram on the right shows the journey a car makes between two sets of traffic lights. Use it to find the approximate distance between the traffic lights



4. The velocity  $v \text{ ms}^{-1}$  of a particle over the first 5 seconds of its motion is represented by the equation  $v = t(5 + 2t)$  where  $t$  is in seconds.
  - a) Copy and complete the table below and from it draw the graph of  $v = t(5 + 2t)$ . Use a

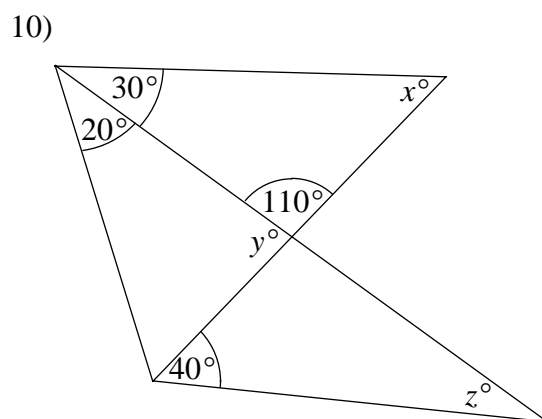
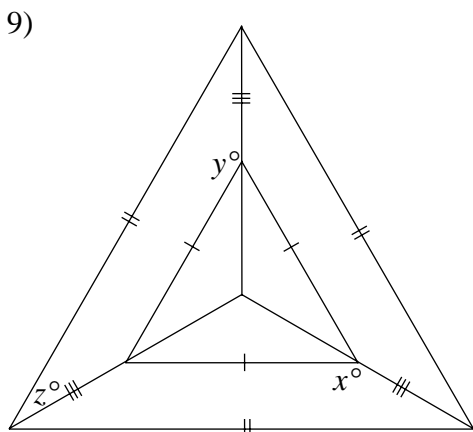
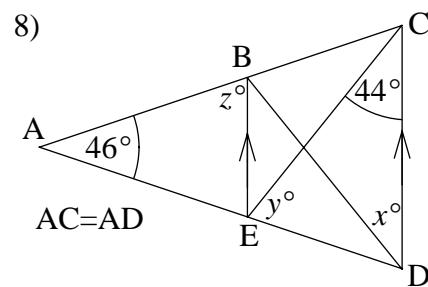
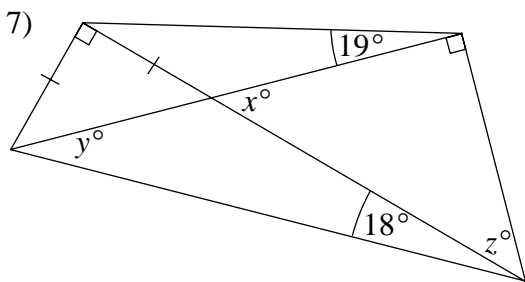
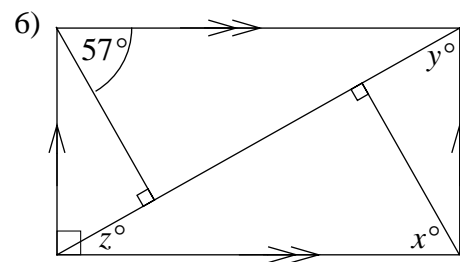
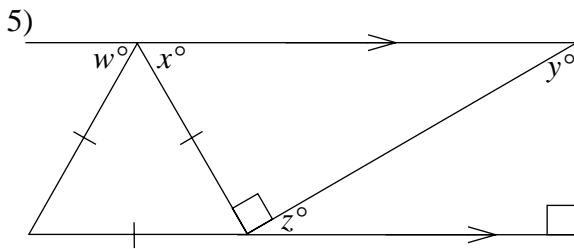
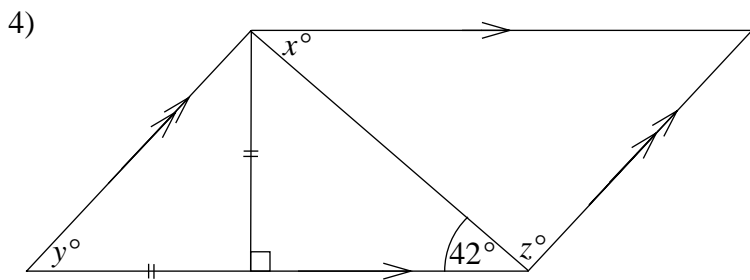
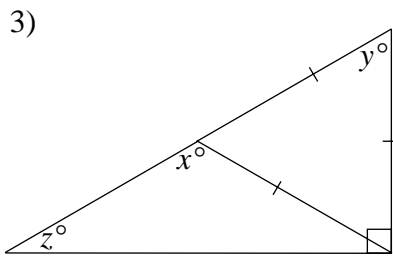
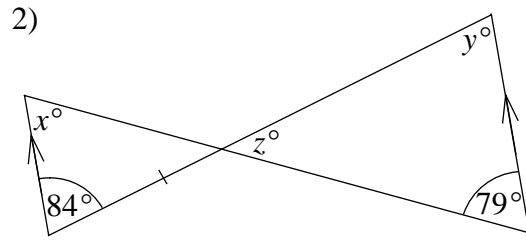
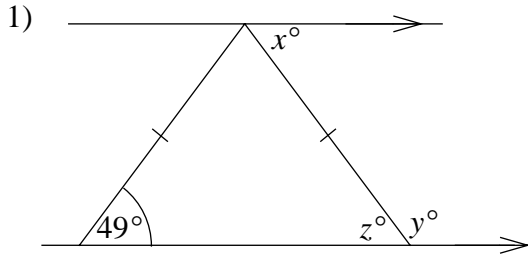
Time $t$	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Velocity $v$											

scale of 2cm to represent 1 second on the horizontal axis and 1cm to represent  $5\text{ms}^{-1}$  on the vertical axis.

- b) Estimate the distance travelled by the particle in the first 3 seconds.
- c) What is the approximate acceleration of the particle when  $t = 3$  seconds?

### 34. Angles and Triangles

In each of the following questions write down the sizes of the unknown angles.

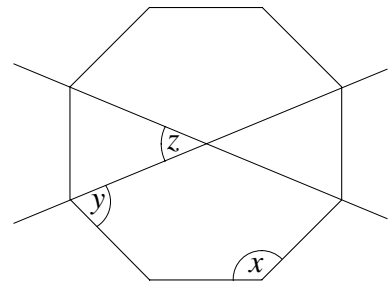




### 35. Regular Polygons

1. a) Calculate the value of the angle  $x$  in this regular octagon.  
 b) What is the size of angle  $y$ ?  
 c) Calculate the size of angle  $z$ .

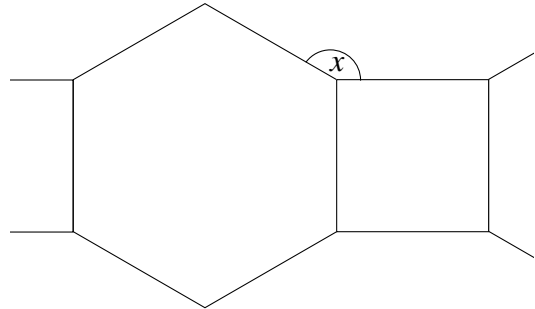
In each, clearly explain how you arrive at your answer.



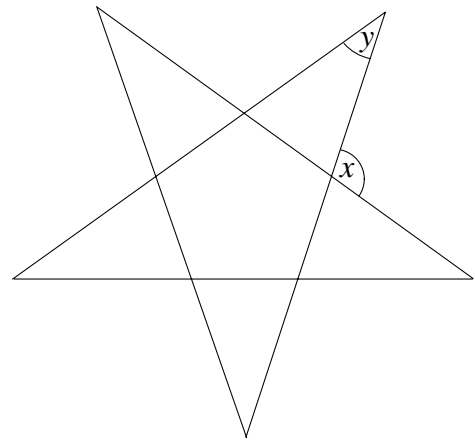
2. What is the sum of all the exterior angles of a polygon?

3. Regular hexagons and squares are put together in a row, as shown in the diagram on the right.

Calculate the size of the angle marked  $x$ . Explain clearly how you arrive at your answer.



4. A five pointed regular star shape is to be cut from a piece of card. In order to draw it accurately it is necessary to calculate the angles  $x$  and  $y$ . Calculate these angles, explaining clearly how you arrive at the answers.

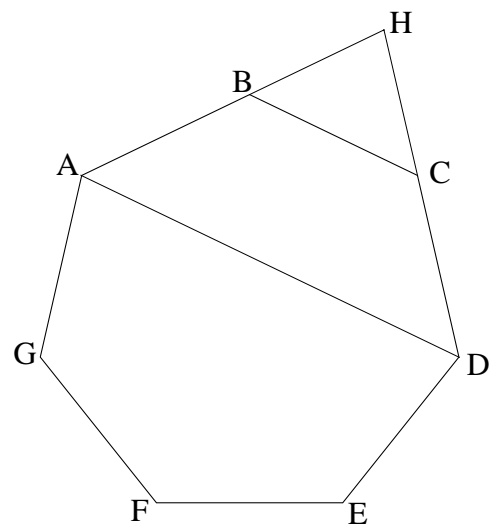


5. What regular shape can be made from a regular hexagon and three regular triangles, all with side lengths of  $x$  centimetres? What will be the length of one side of the new shape?

6. In the regular heptagon shown on the right AB and DC are produced to meet at H.

a) Calculate the sizes of angles BCD and BHC, in each case explaining how you arrive at your answer.

b) Prove that BC is parallel to AD.

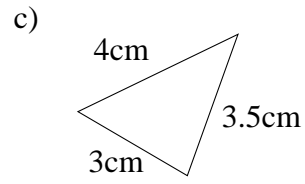
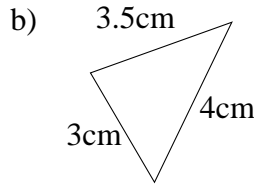
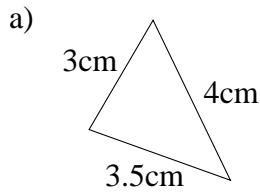


7. Three regular polygons fit together around a point. One is a triangle and one is an octagon. Calculate the number of sides the third shape has.

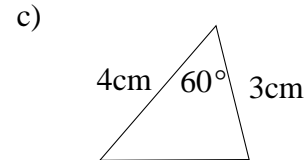
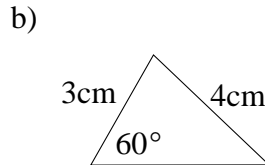
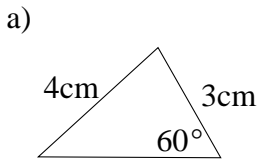
### 36. Congruent Triangles

In questions 1 to 4, say whether all, two or none of the triangles are congruent.

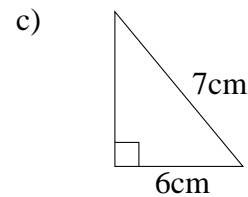
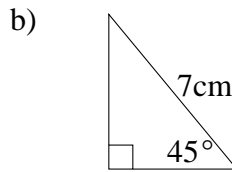
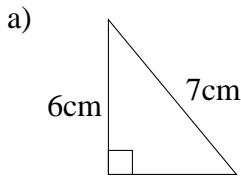
1.



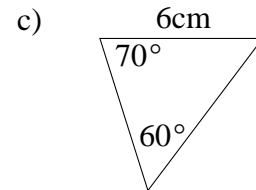
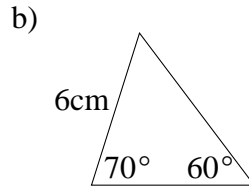
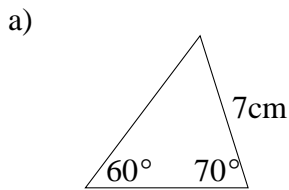
2.



3.

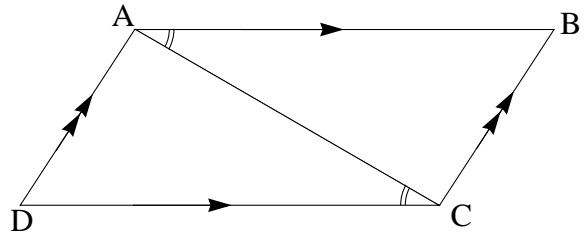


4.

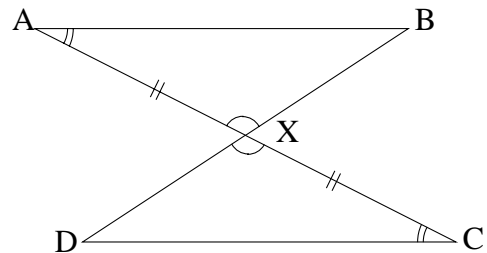


In questions 5 and 6 the triangles indicated are congruent. In each case explain why they are congruent.

5.  $\triangle ABC$  and  $\triangle ADC$



6.  $\triangle ABX$  and  $\triangle CDX$



7. Which, if any, of the following statements are true for the triangles on the right

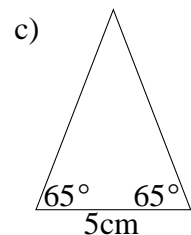
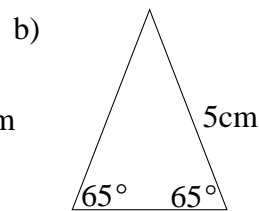
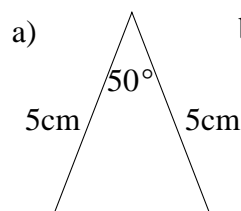
(i) Triangles a and b are congruent

(ii) Triangles a and c are congruent

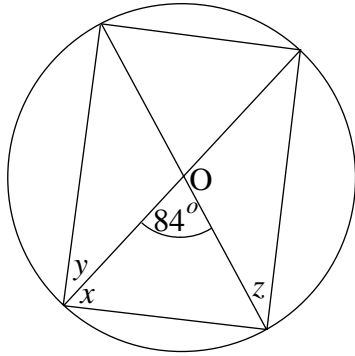
(iii) Triangles b and c are congruent

(iv) All three triangles are congruent

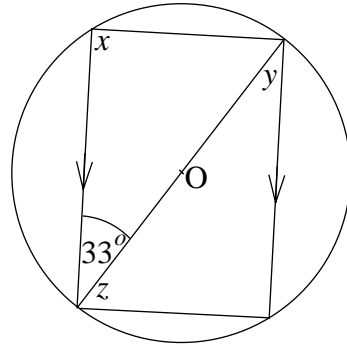
(v) None of the triangles are congruent.



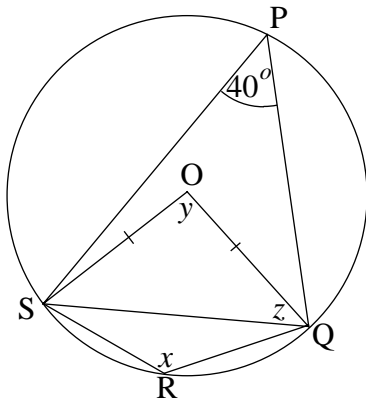
### 37. Geometry of a circle 1



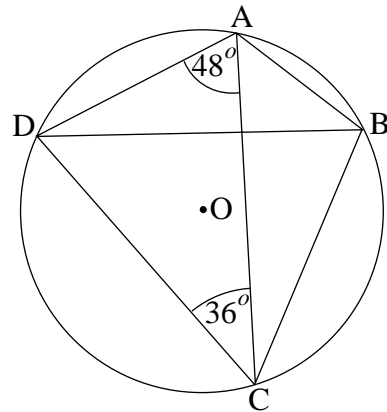
1) Calculate the sizes of the angles  $x$ ,  $y$  and  $z$ .



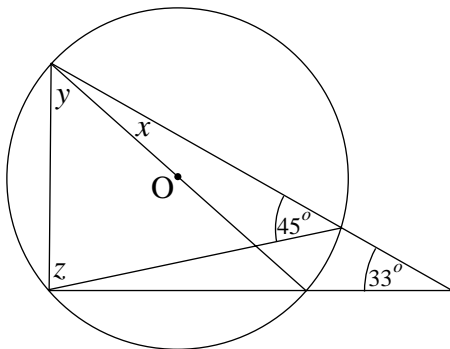
2) Calculate the sizes of the angles  $x$ ,  $y$  and  $z$ .



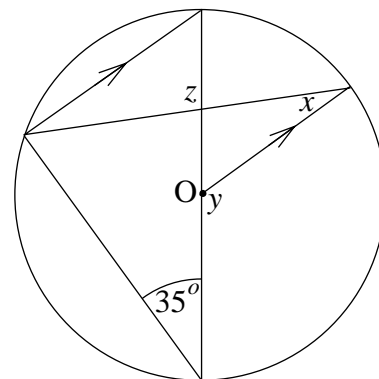
3) O is the centre of the circle. Lines OQ, and OS are equal in length. Calculate the sizes of angles  $x$ ,  $y$  and  $z$ .



4) Calculate the sizes of angles ADC, ABC and AOD.

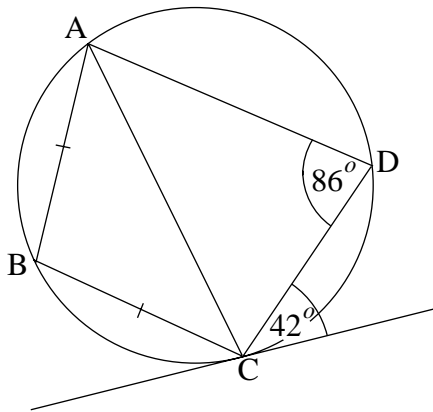


5) Calculate the sizes of angles  $x$ ,  $y$  and  $z$ .

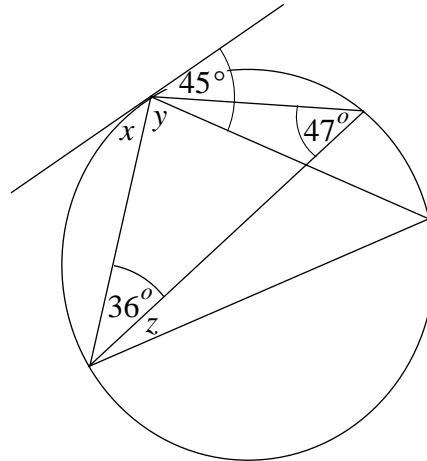


6) Calculate the values of the angles  $x$ ,  $y$  and  $z$ .

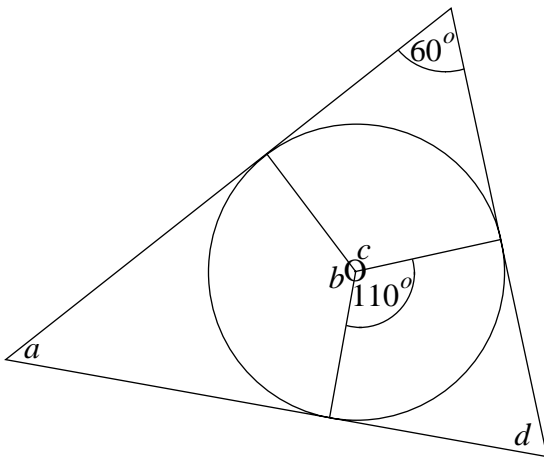
### 38. Geometry of a circle 2



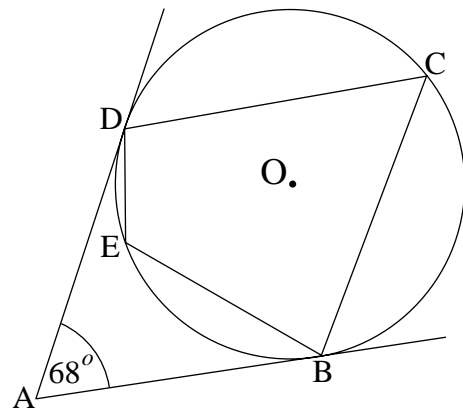
1) If  $AB=BC$ , calculate the sizes of the angles  $ABC$ ,  $ACB$ ,  $ACD$  and  $DAC$ .



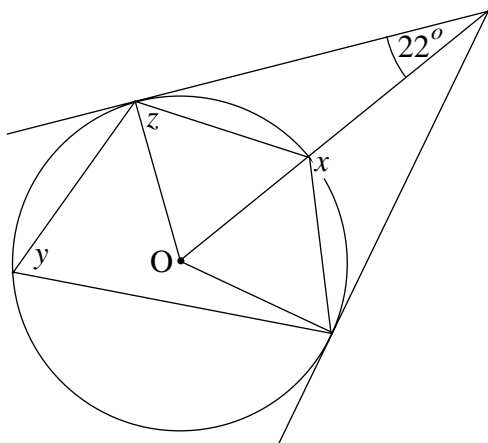
2) Find the values of the angles  $x$ ,  $y$  and  $z$ .



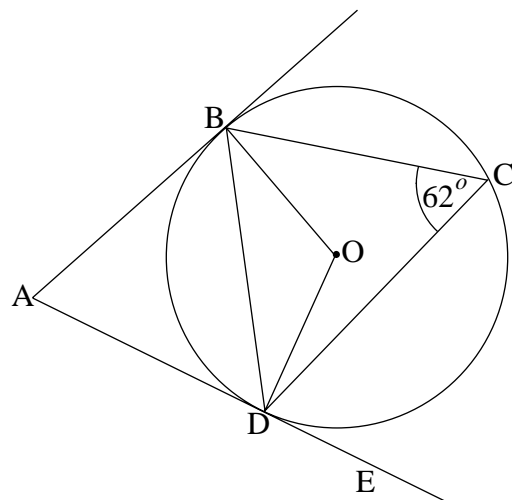
3) O is the centre of the circle. Calculate the sizes of the angles  $a$ ,  $b$ ,  $c$  and  $d$ .



4) Calculate the angles  $DEB$ ,  $BCD$ , and  $DBO$ .



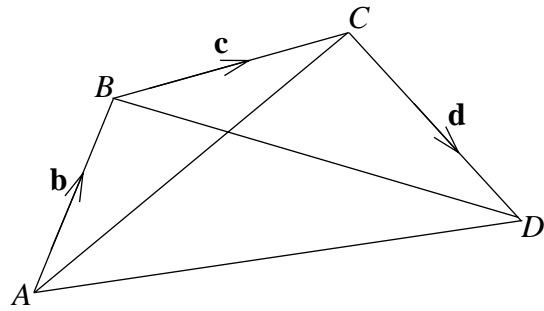
5) Calculate the angles  $x$ ,  $y$  and  $z$ .



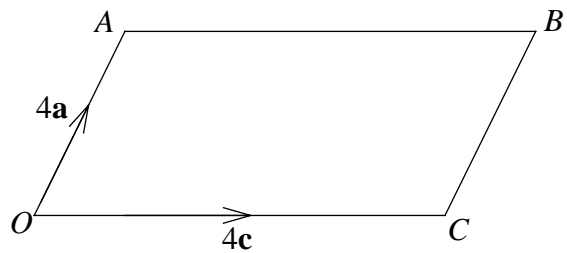
6) Calculate the angles  $BDA$ ,  $BOD$ ,  $BAD$  and  $DBO$ .

## 39. Vectors 1

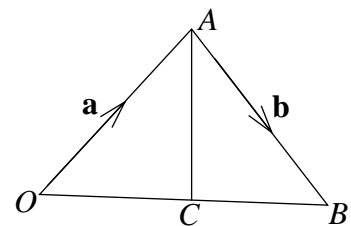
1. In the quadrilateral,  $\vec{AB}$ ,  $\vec{BC}$ , and  $\vec{CD}$  are represented by the vectors  $\mathbf{b}$ ,  $\mathbf{c}$ , and  $\mathbf{d}$ . Find, in terms of  $\mathbf{b}$ ,  $\mathbf{c}$ , and  $\mathbf{d}$   $\vec{AC}$ ,  $\vec{AD}$ , and  $\vec{BD}$
2.  $\vec{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  and  $\vec{BC} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$  calculate  $\vec{AC}$  in bracket form.
3. Express  $\vec{AB} + \vec{BC} + \vec{CD}$  in its simplest form.



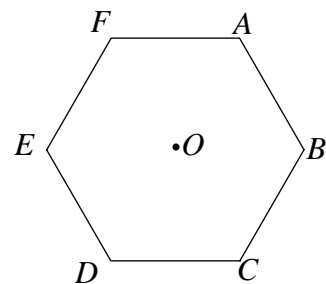
4.  $OABC$  is a parallelogram. If  $\vec{OA} = 4\mathbf{a}$  and  $\vec{OC} = 4\mathbf{c}$ . Find, in terms of  $\mathbf{a}$  and  $\mathbf{c}$  (a)  $\vec{AC}$  (b)  $\vec{AP}$  where  $P$  is the mid point of  $AC$  (c)  $\vec{OP}$  (d) If  $X$  is the mid point of  $CB$  find  $\vec{PX}$ . (e) What is the geometrical relationship between  $PX$  and  $OC$ ?



5. In triangle  $OAB$ ,  $\vec{OA} = 6\mathbf{a}$  and  $\vec{OB} = 9\mathbf{b}$ . Point  $P$  is on  $AB$  such that  $AP = \frac{1}{2}PB$ . Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$  (a)  $\vec{AB}$  (b)  $\vec{AP}$  (c)  $\vec{OP}$ .
6.  $OABC$  is a triangle with point  $C$  half way along  $OB$ .  $\vec{OA} = \mathbf{a}$  and  $\vec{AB} = \mathbf{b}$ . Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$  (a)  $\vec{OB}$  (b)  $\vec{OC}$  (c)  $\vec{AC}$

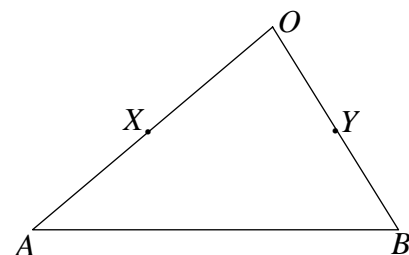


7.  $ABCDEF$  is a regular hexagon with centre  $O$ .  $\vec{OA} = \mathbf{a}$  and  $\vec{OB} = \mathbf{b}$ . Express in terms of  $\mathbf{a}$  and  $\mathbf{b}$  the vectors  $\vec{OD}$ ,  $\vec{OE}$ ,  $\vec{DA}$ ,  $\vec{BC}$ ,  $\vec{OC}$  and  $\vec{CF}$ .



8. (a) If  $\vec{AB} = 3\mathbf{a}$  and  $\vec{CD} = 6\mathbf{a}$  write down the geometrical relationship between  $\vec{AB}$  and  $\vec{CD}$ . (b) What is the geometrical relationship between vectors  $4\mathbf{a} + 2\mathbf{b}$  and  $6\mathbf{a} + 3\mathbf{b}$ ?

9.  $OAB$  is a triangle with point  $X$  half way along  $OA$  and  $Y$  half way along  $OB$ . If  $\vec{OA} = 2\mathbf{a}$  and  $\vec{OB} = 2\mathbf{b}$ . Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , (a)  $\vec{AB}$  (b)  $\vec{XY}$  and (c) show that  $XY$  is parallel to  $AB$ .

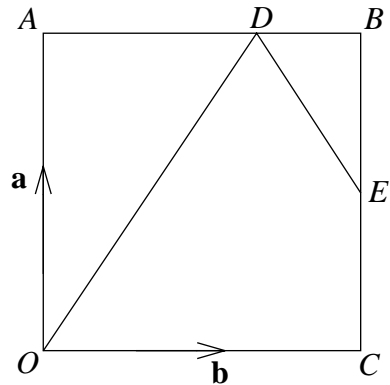


10. In a rectangle  $ABCD$ ,  $\vec{AB} = \mathbf{a}$  and  $\vec{BC} = \mathbf{b}$ . Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$  the vectors  $\vec{AC}$  and  $\vec{BD}$ .

### 40. Vectors 2

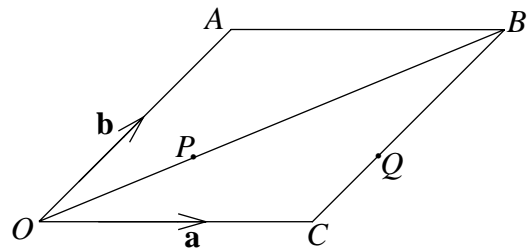
1.  $OABC$  is a square.  $\vec{OA} = \mathbf{a}$  and  $\vec{OC} = \mathbf{b}$ . Point  $D$  cuts the line  $AB$  in the ratio 2:1.

- (a) Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$  the vector that represents  $\vec{AD}$ .
- (b) If point  $E$  is halfway along  $BC$ , find the vector that represents  $\vec{DE}$ .
- (c) Using your answers to parts a and b show that the vector  $\vec{OE}$  is represented by  $\mathbf{b} + \frac{1}{2}\mathbf{a}$ .



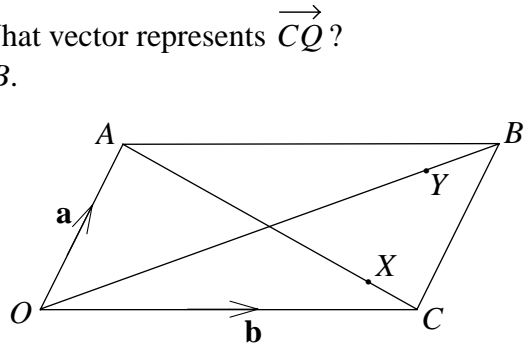
2.  $OABC$  is a rhombus with  $\vec{OA}$  represented by  $\mathbf{b}$  and  $\vec{OC}$  represented by  $\mathbf{a}$ .

- (a) What is the vector  $\vec{OB}$  represented by?
- (b) What vector represents  $\vec{OP}$  if point  $P$  divides the line  $OB$  in the ratio 1:2?
- (c) Point  $Q$  divides the line  $CB$  in the ratio 1:2. What vector represents  $\vec{CQ}$ ?
- (d) Show that the line  $PQ$  is parallel to the line  $AB$ .



3. In the parallelogram  $OABC$ , point  $Y$  cuts the line  $OB$  in the ratio 5:1. Point  $X$  cuts the line  $AC$  in the ratio 5:1 also. Write down in terms of  $\mathbf{a}$  and  $\mathbf{b}$

- (a)  $\vec{OB}$  (b)  $\vec{YB}$  (c)  $\vec{AC}$  (d)  $\vec{XC}$  (e)  $\vec{XY}$ .
- (f) What is the geometric relationship between lines  $XY$  and  $CB$ ?



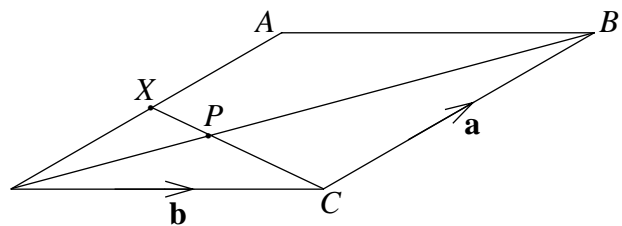
4.  $OABC$  is a rhombus.

$\vec{CB} = \mathbf{a}$  and  $\vec{OC} = \mathbf{b}$ .

Point  $X$  is halfway along  $OA$ .

Point  $P$  cuts  $XC$  in the ratio 1:2. Write down, in terms of  $\mathbf{a}$  and  $\mathbf{b}$  vectors for

- (a)  $\vec{OX}$  (b)  $\vec{XC}$  (c)  $\vec{OB}$
- (d)  $\vec{XP}$  (e)  $\vec{OP}$  (f) How far along  $OB$  is point  $P$ ?

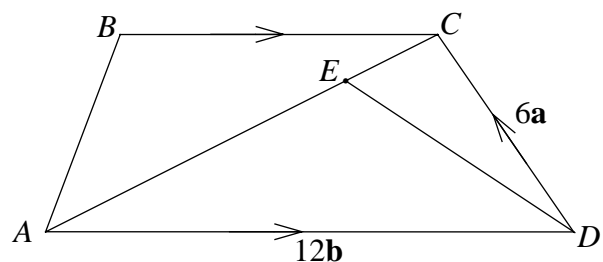


5.  $ABCD$  is a trapezium with  $BC$  parallel to  $AD$  and  $\frac{3}{4}$  of its length.

$\vec{AD} = 12\mathbf{b}$   $\vec{DC} = 6\mathbf{a}$ .

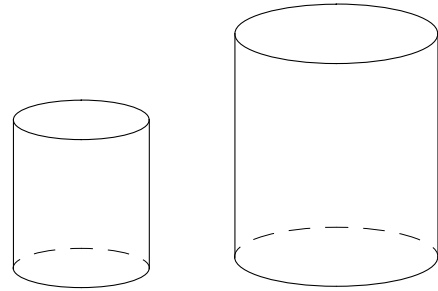
Point  $E$  cuts the line  $AC$  in the ratio 5:1. Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$

- (a)  $\vec{AC}$  (b)  $\vec{DE}$  (c)  $\vec{AB}$

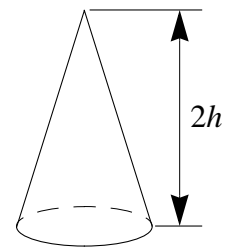


## 41. Similar Shapes

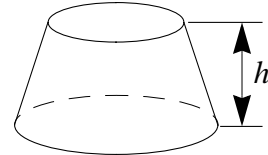
1. Two hollow cylinders are similar in shape. One is 12cm tall and the other is 18cm tall. Calculate the ratio of
- the areas of their ends
  - their volumes.
  - If the smaller one has a volume of  $100 \text{ cm}^3$ , what is the volume of the larger one?



2. A cone of height  $3h$  is cut into two parts to make a smaller cone of height  $2h$ .
- What is the ratio of the volumes of the two parts?
  - If the surface area of the top part is  $200 \text{ cm}^2$ , what was the surface area of the original cone?

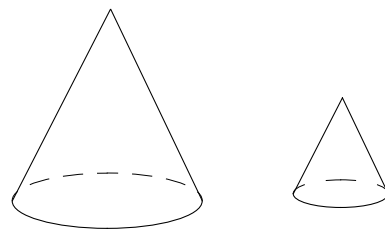


3. Two similar blocks of metal made from the same material have a total surface area of  $25 \text{ cm}^2$  and  $64 \text{ cm}^2$ . If the smaller one has a mass of 30 grams, what is the mass of the larger one?

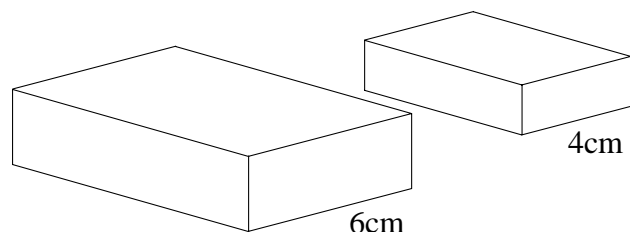


4. A ball has a diameter of 8cm and weighs 200 grams. Calculate the weight of a ball of 10cm diameter made from the same material.
5. A cylindrical can of height 15cm holds one litre of orange juice. What height, to the nearest mm, must a similar can be if it holds 500ml?
6. Two similar cones have heights of 100cm and

50cm. If the volume of the smaller one is  $1000 \text{ cm}^3$ , calculate the volume of the larger one.



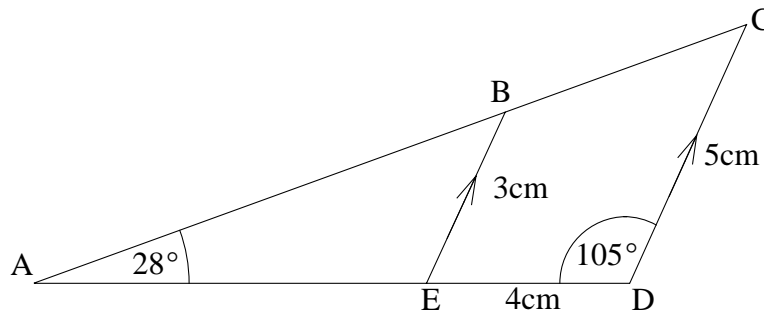
7. Two similar boxes have corresponding sides of 6cm and 4cm.
- If the surface area of the smaller box is  $50 \text{ cm}^2$ , what is the surface area of the larger box?
  - If the volume of the larger box is  $50 \text{ cm}^3$ , what is the volume of the smaller box?



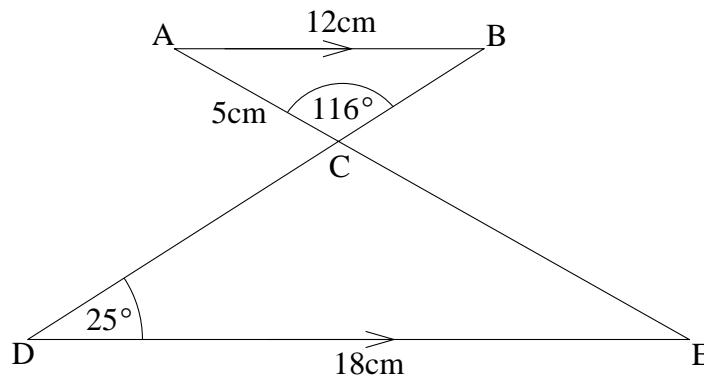
8. Two similar cornflakes packets have widths of 25cm and 20cm. Cornflakes are sold in packets of 750g, 500g and 350g. If the 25cm packet holds 750g, how much does the 20cm packet hold?

### 42. Similarity

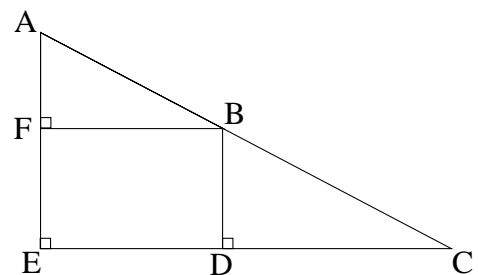
1. In the triangle ABCDE,  $BE = 3\text{cm}$ ,  $CD = 5\text{cm}$ , angle  $CDE = 105^\circ$  and angle  $BAE = 28^\circ$ . Line BE is parallel to line CD.



- a) (i) Calculate the size of angle DEB  
 (ii) Explain your answer.  
 b) Calculate the size of AE.
2. In the diagram,  $AB = 12\text{cm}$ ,  $DE = 18\text{cm}$ ,  $AC = 5\text{cm}$ , angle  $ACB = 116^\circ$  and angle  $CDE = 25^\circ$ . Line AB is parallel to line DE.

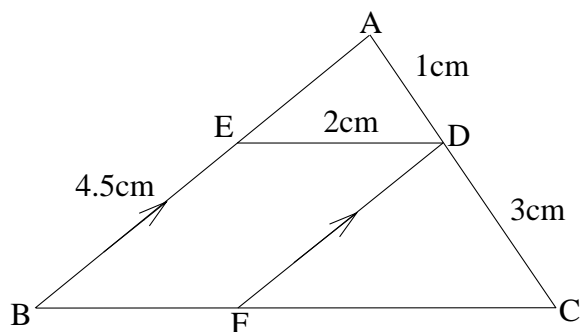


- a) (i) Calculate the size of angle DEC.  
 (ii) Explain how you get your answer.  
 b) Calculate the length of CE
3. In the triangle,  $AC = 20\text{cm}$ ,  $DC = 10\text{cm}$  and  $FB = 6\text{cm}$ .



- a) Calculate the length of AB.  
 b) If  $AF = 4.5\text{ cm}$ , calculate the length of FE.

4. In the triangle shown below,  $AD = 1\text{cm}$ ,  $DC = 3\text{cm}$ ,  $ED = 2\text{cm}$  and  $BE = 4.5\text{cm}$ . Line AB is parallel to line DF.



- a) Calculate the lengths of FC and AE.  
 b) What is the ratio of the areas of triangle AED to that of triangle ABC.

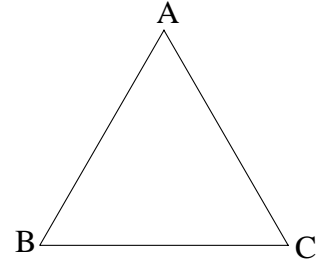


### 43. Bearings

1. An aeroplane sets off on a bearing of  $N 28^\circ E$  ( $028^\circ$ ) but after some time has to turn back to the airport it came from. On what bearing must it travel?
2. A ship sets sail on a bearing of  $S 17^\circ E$  ( $163^\circ$ ). It then turns through an angle of  $90^\circ$  anticlockwise. What is its new bearing?
3. ABC is an equilateral triangle with line BC pointing due east.

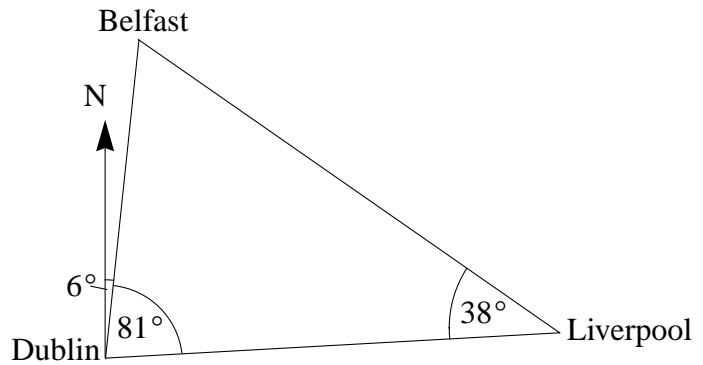
Write down the following bearings:

- a) A from B
- b) C from A
- c) A from C



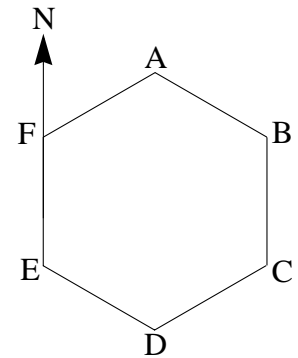
4. The diagram shows the approximate relative positions of Belfast, Dublin and Liverpool. Calculate the bearings of:

- a) Belfast from Dublin
- b) Belfast from Liverpool
- c) Liverpool from Belfast.



5. The diagram shows a regular hexagon ABCDEF with one side pointing due north. Calculate the bearings of:

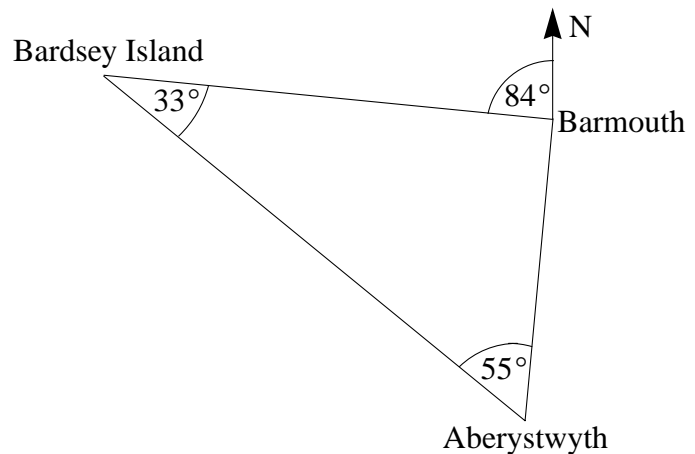
- a) A from F
- b) C from B
- c) D from C
- d) E from B



6. The diagram shows the approximate relative positions of Bardsey Island, Barmouth and Aberystwyth.

Calculate the bearings of:

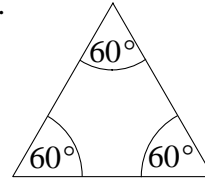
- a) Bardsey Island from Aberystwyth
- b) Barmouth from Aberystwyth
- c) Bardsey Island from Barmouth.



### 44. Constructions

In the questions below, use only a ruler and a compass, not a protractor.

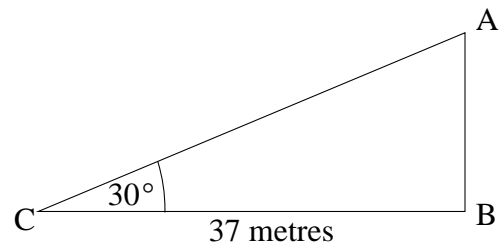
1. Construct an angle of  $60^\circ$  and bisect it to make an angle of  $30^\circ$ .
2. Construct an angle of  $90^\circ$  and bisect it to make an angle of  $45^\circ$ .
3. Construct this equilateral triangle, with sides of 5cm.



4. Construct this rectangle

5. Draw a line 12cm long and bisect it by construction.

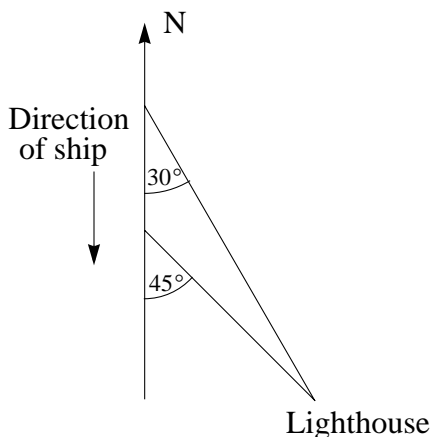
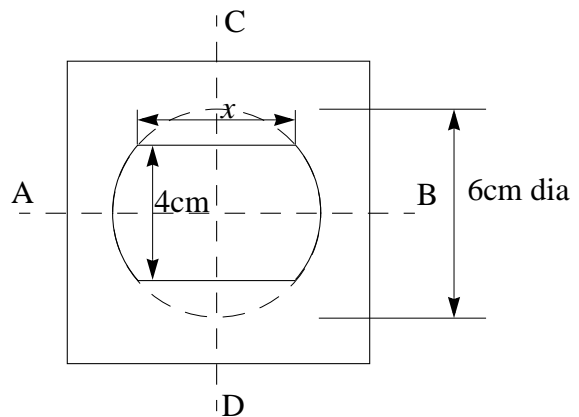
6. Joanna wants to measure the height of a tower. She measures out a distance from the bottom of the tower (BC) until the angle between the ground and the top of the tower is  $30^\circ$ . She measures BC to be 37 metres. By making a scale drawing and letting 1cm represent 5 metres, estimate the height of the tower.



7. The diagram shows the sketch of an 8cm square metal plate with a hole at the centre. The shape is symmetric about the lines AB and CD.

Make an accurate drawing of the hole.

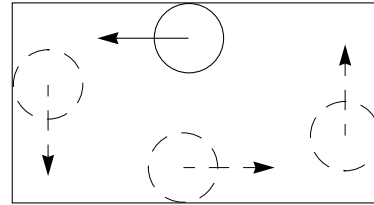
What is the length of dimension  $x$ ?



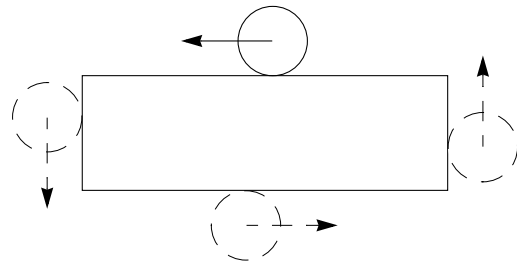
8. A ship is sailing due south when it detects a light house at an angle of  $30^\circ$  from its direction of travel. It carries on for a further 1200 metres and now finds that the lighthouse is at an angle of  $45^\circ$  to its direction of travel. Draw an accurate diagram to a scale of 1cm to represent 200 metres. By measuring on your diagram, estimate the closest distance the ship will be to the lighthouse if it continues on this course.

### 45. Loci

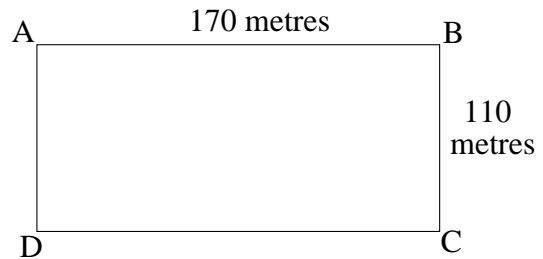
1. A wheel of diameter 3cm rotates around the inside of a rectangle measuring 8cm by 11cm. Draw accurately the locus of the centre of the wheel.



2. A wheel of diameter 3cm rotates around the outside of a rectangle measuring 10cm by 4cm. Draw the locus of the centre of the wheel.

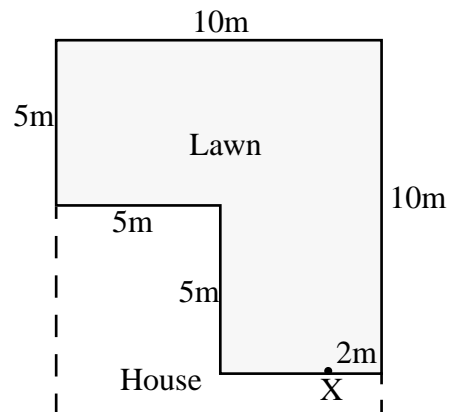


3. A rectangular field ABCD is to have a pipe line laid underneath it. The pipe enters the field through the side AD, is parallel to the side AB and 40 metres from it. After some distance its route turns at a point X towards corner C. Between X and C it is always equidistant from the sides BC and CD. It finally leaves the field by the corner C.

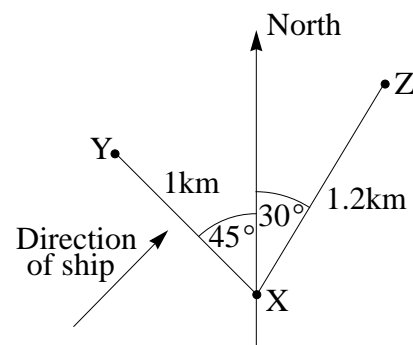


Copy the plan of the field, using a scale of 1cm to represent 20 metres. On the plan clearly indicate the path of the pipe line. Measure and record the distance BX.

4. The diagram shows the plan of a garden lawn. Draw an accurate diagram using the scale of 1cm to represent 1 metre. Point X represents an electrical socket on the side of the house. An electric lawn mower has a cable attached to it allowing it to travel up to 9 metres from the socket. Show clearly on the diagram the area of the lawn which can be mown.

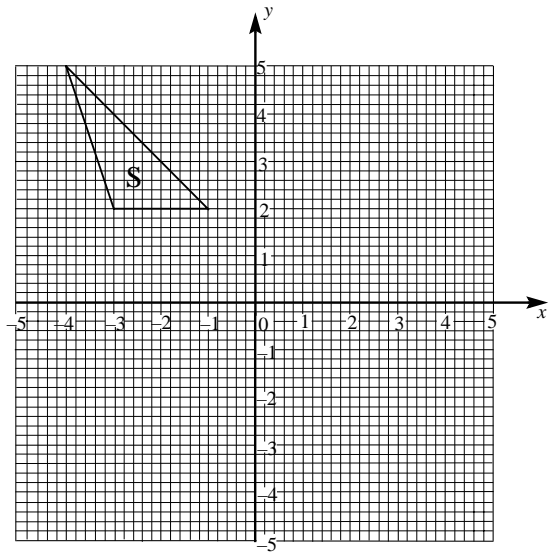


5. A ship sails between two rocks X and Y. The ship is at all times equidistant from both rocks. At point P, when it is the same distance from rocks Y and Z, it alters course, keeping to a path which is equidistant from Y and Z. Draw an accurate diagram to a scale of 5cm to represent 1km to show the route of the ship. What is the distance XP?



### 46. Transformations 1

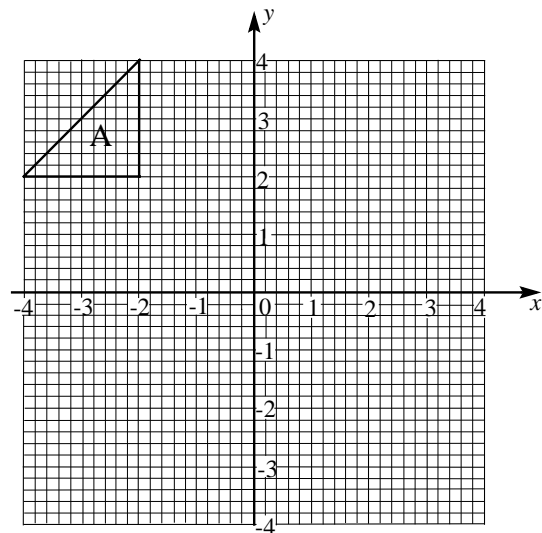
1. a) The triangle S is reflected across the line  $y = 0$  to triangle T. Copy the diagram and show triangle T. What are the co-ordinates of triangle T?
- b) Triangle T is reflected across the line  $x = 0$  to triangle U. Draw triangle U and write down its co-ordinates.
- c) What is the single transformation that transforms triangle U onto triangle S?



2. Triangle A has vertices at  $(-2,2)$ ,  $(-4,2)$  and  $(-2,4)$ . Copy the diagram and draw each of the following transformations.

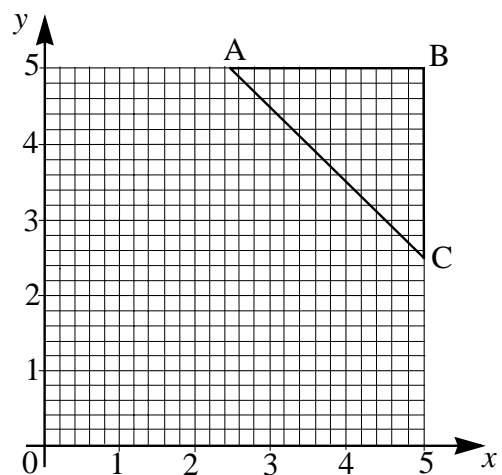
In each case write down their co-ordinates.

- a) A rotation of triangle A of  $90^\circ$  clockwise about the origin. Label this B.
- b) A reflection of A about the line  $y = x$ . Label this C.
- c) An enlargement of A with a scale factor of  $\frac{1}{2}$  through the origin  $(0,0)$ . Label this D.
- d) What single transformation will transform D back into A?



3. Triangle A, B, C is transformed into triangle  $A_1, B_1, C_1$  by an enlargement of scale factor  $\frac{2}{5}$  through the origin  $(0,0)$ .

- a) Draw triangles A, B, C and  $A_1, B_1, C_1$ .
- b) Write down the co-ordinates of triangle  $A_1, B_1, C_1$ .
- c)  $A_1, B_1, C_1$  is transformed onto A, B, C. What single transformation will do this?



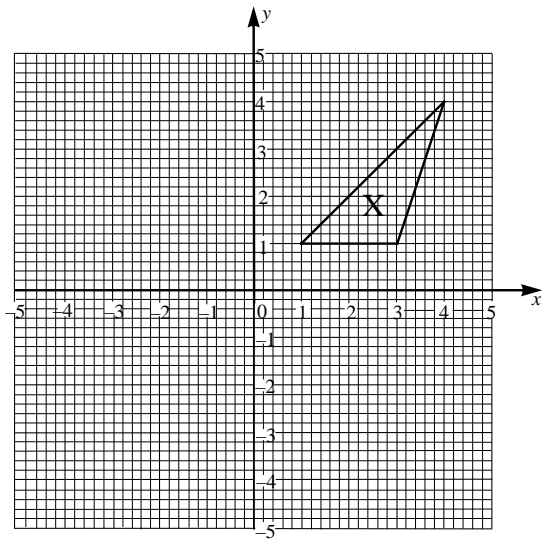
### 47. Transformations 2

1. Triangle X is rotated  $180^\circ$  anticlockwise about the origin to triangle Y.

a) Draw triangles X and Y and write down the co-ordinates of Y.

b) Triangle Y is translated by the vector  $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$  to triangle Z. Draw triangle Z and write down its co-ordinates.

c) What single transformation will transform triangle X onto Z?

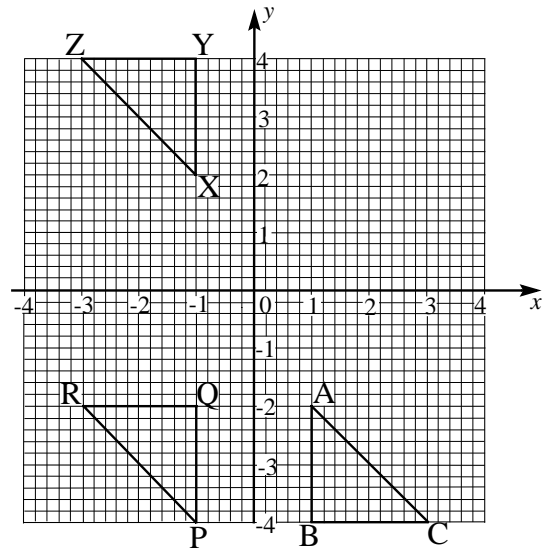


2. a) Triangle ABC is transformed onto triangle XYZ. What single transformation will do this?

b) Triangle XYZ is transformed onto triangle PQR. What single transformation will do this?

c) If triangle PQR is rotated through  $180^\circ$  about the point  $(-1, 0)$  to triangle STU, what are the co-ordinates of the new shape?

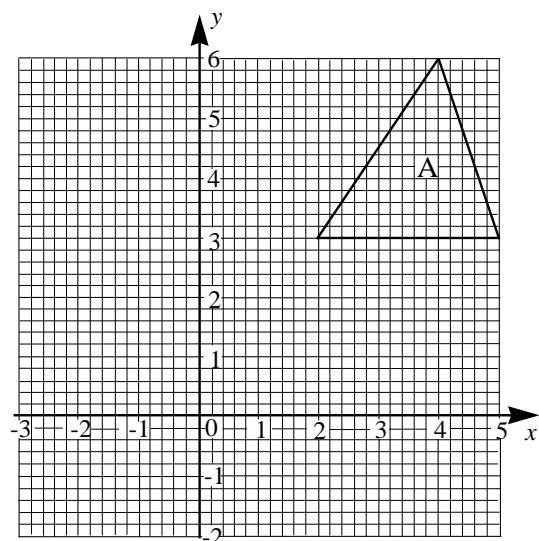
d) What single transformation will transform triangle XYZ onto triangle STU?



3. a) Triangle A is reflected about the line  $y = 2$  to triangle B. Draw triangles A and B and write down the co-ordinates of triangle B.

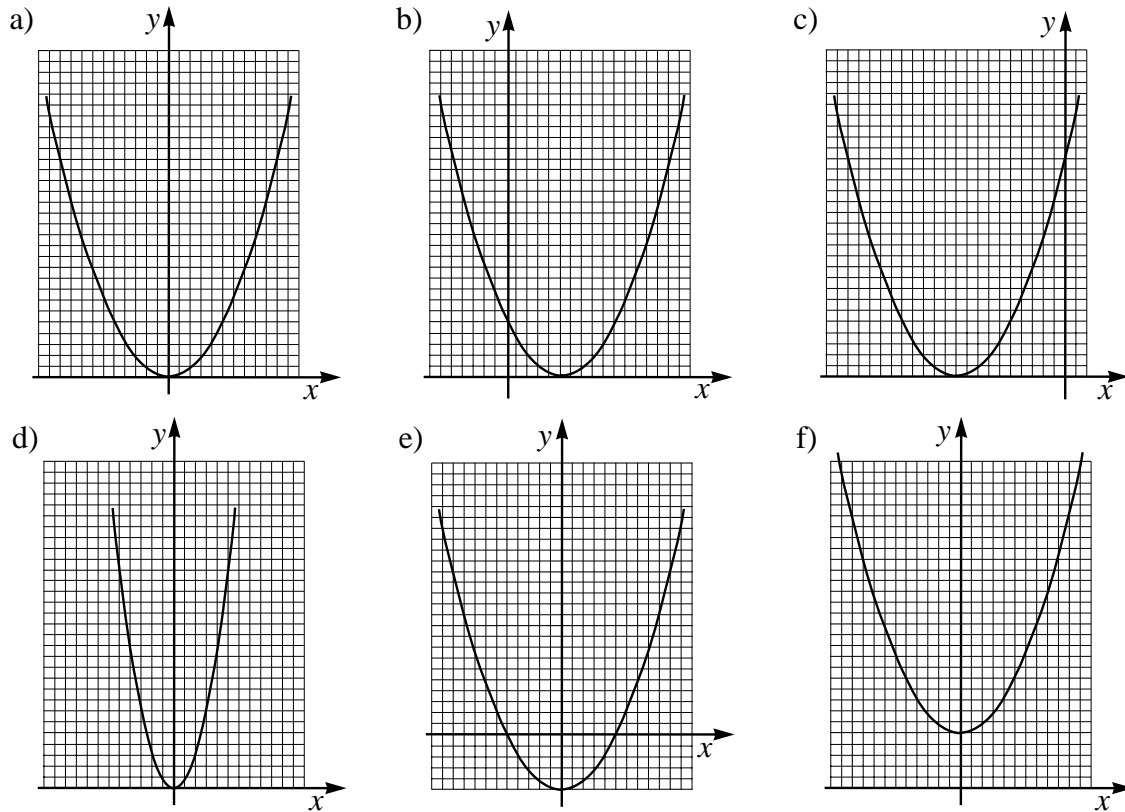
b) Triangle B is rotated  $90^\circ$  clockwise about the point  $(1, 2)$  onto triangle C. Draw triangle C and write down its co-ordinates.

c) What single transformation will transform triangle A onto triangle C?



### 48. Transformations of Graphs

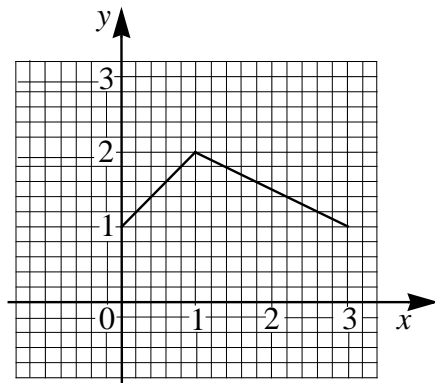
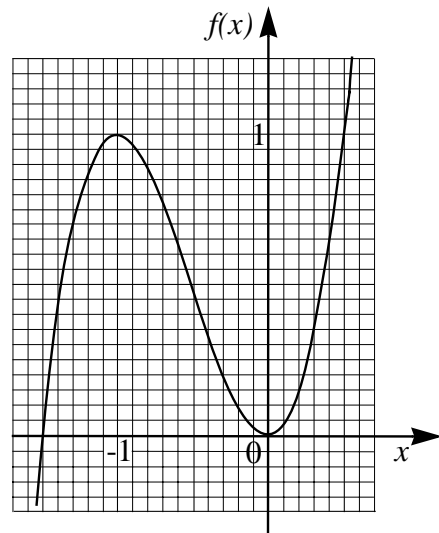
1. Diagram 'a' below shows a sketch of the graph  $y = x^2$ . The other graphs show (i)  $y = x^2 + 1$ , (ii)  $y = x^2 - 1$ , (iii)  $y = (x - 1)^2$ , (iv)  $y = (2x)^2$ , and (v)  $y = (x + 2)^2$  but not in that order. Match the diagram to the equation.



2. Explain clearly each of the transformations in question 1.  
 3. The diagram on the right shows the graph of the

function  $f(x) = 2x^3 + 3x^2$ .

- a) If this graph is reflected in the  $x$  axis what will its equation be?  
 b) Sketch the graph.  
 c) If  $f(x)$  is transformed into  $f(-x)$  write down its equation and explain the transformation.  
 4. The function  $f(x)$  is defined for  $0 < x < 3$  in the diagram below. Sketch the functions  
 a)  $y = f(x - 2)$  and b)  $y = f(2x)$



### 49. Matrix Transformations

1. By first drawing triangle A(1,1), B(3,1), C(3,4) use each of the following matrices to transform it. State the transformation each represents.

$$\text{a) } \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \text{c) } \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{d) } \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2. A transformation is given by the matrix  $\mathbf{P} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ . Find  $\mathbf{P}^{-1}$ , the inverse of  $\mathbf{P}$  and use it

to find the co-ordinates of the point whose image is (1,3).

3. A' is the image of triangle A after it has been reflected about the y axis. A'' is the image of the triangle A' after it has been rotated  $180^\circ$  about the origin (0,0).

a) If triangle A has co-ordinates (1,1), (2,1), and (1,3) draw triangle A and the images A' and A''.

b) Find the  $2 \times 2$  matrix  $\mathbf{M}$  associated with the transformation A to A'.

c) Find the  $2 \times 2$  matrix  $\mathbf{N}$  associated with the transformation A' to A''.

d) Calculate the matrix product  $\mathbf{NM}$  and state the single transformation which is defined by this matrix.

4. Triangle T has co-ordinates (1,1), (3,2), and (1,2). Triangle T' is the image of triangle T when rotated through  $180^\circ$  about the origin.

a) What is the matrix  $\mathbf{M}$  associated with the transformation of T to T'?

Triangle T' is further transformed to T'' by the matrix  $\mathbf{N} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

b) What transformation does matrix  $\mathbf{N}$  perform?

c) State the single transformation which is defined by the matrix product  $\mathbf{NM}$  and write down this matrix.

5. Triangle A(1,1), B(2,2), C(1,4) is transformed onto triangle  $A_1B_1C_1$  by the matrix  $\mathbf{M}$

$$\text{where } \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Triangle  $A_1B_1C_1$  is transformed by the matrix  $\mathbf{N}$  where  $\mathbf{N} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$  onto the triangle

$A_1B_1C_2$ .

a) Find the co-ordinates of  $A_1$ ,  $B_1$ ,  $C_1$  and  $C_2$  and draw the three triangles.

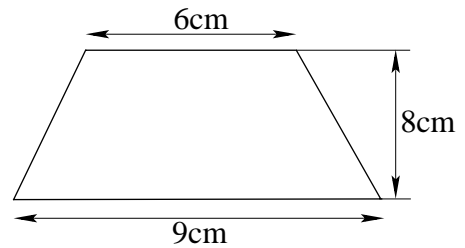
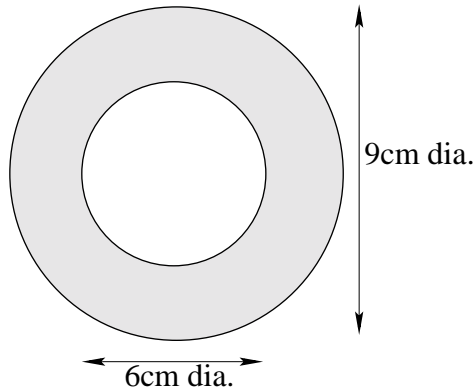
b) What single transformation will transform triangle ABC onto triangle  $A_1B_1C_2$ ?

c) By calculating the matrix  $\mathbf{W} = \mathbf{NM}$  show that this matrix represents the transformation found in part 'b'.

d) Calculate  $\mathbf{W}^{-1}$ , the inverse of  $\mathbf{W}$ . Use it to find the co-ordinates of the point whose image is (3,-5).

### 50. Area and Perimeter

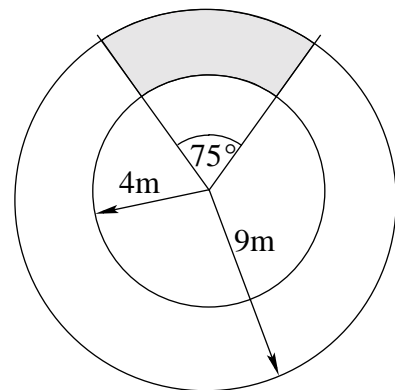
1. Calculate the area of the trapezium on the right.
2. Calculate the area of the shaded part of the ring below.



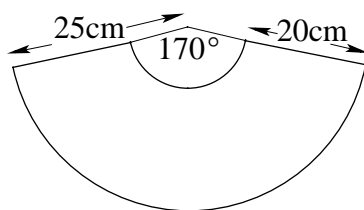
3. Calculate the area and arc length of the following sectors where  $r$  is the radius of the circle and  $\theta$  is the angle at the centre of the circle.
 

a) $r = 6\text{cm}$ $\theta = 30^\circ$	b) $r = 12\text{cm}$ $\theta = 140^\circ$	c) $r = 16.5\text{cm}$ $\theta = 200^\circ$	d) $r = 16.75\text{cm}$ $\theta = 195^\circ$
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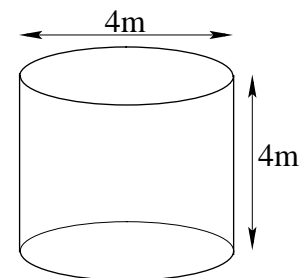
4. The shaded part of the diagram shows a flower garden in the shape of an arc. It has an inside radius of 4 metres and an outside radius of 9 metres. a) Calculate the area of the garden. b) If it is completely surrounded by an edging strip, calculate its length.



5. A conical lamp shade is to be made from the piece of fabric shown in the diagram. a) Calculate its area. b) If a fringe is sewn onto its bottom edge, what length will it be?



6. A water tank is in the form of a cylinder of 4 metres diameter and 4 metres high, open at the top. It is to be made from sheet steel and painted with three coats of paint, both inside and outside. Calculate a) the total area of steel needed and b) the number of tins of paint needed if one tin is sufficient to cover  $28\text{m}^2$
7. The ceiling of a church is in the shape of a hemisphere. If its radius is 8 metres, calculate its surface area.
8. The diameter of the earth is approximately 12734 kilometres. Calculate its surface area.





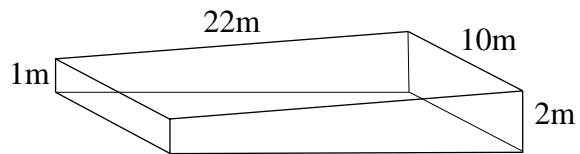
## 51. Volume

1. Calculate the volume of a 12cm square based pyramid with a height of 20cm.
2. Calculate the height of a 4cm square based pyramid whose volume is  $40\text{cm}^3$
3. A rolling pin is in the shape of a cylinder with hemispherical ends. Its total length is 40cm

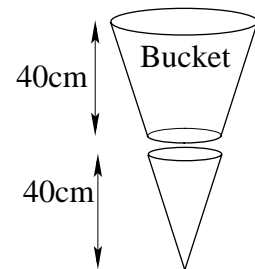


and its diameter is 5cm. Calculate a) its volume and b) its weight if  $1\text{cm}^3$  of wood weighs 0.75 grammes.

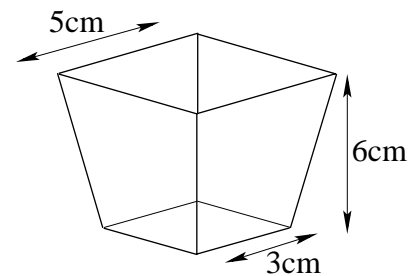
4. A marble paperweight is in the shape of a hemisphere of radius 4cm. Calculate its volume. A supplier packs them into boxes of 50. Calculate their weight, correct to the 0.1kg, if  $1\text{cm}^3$  of marble weighs 2.7 grammes.
5. A metal sphere of diameter 10cm is lowered into a cylindrical jar of 16cm height and 12cm diameter which contains water to a depth of 10cm. How far up the side of the jar will the water rise?
6. The diagram shows a swimming pool. It measures 22 metres long by 10 metres wide. It is 1 metre deep at the shallow end and 2 metres deep at the other end. Calculate the amount of water it will hold, in litres.



7. A bucket is made by cutting a cone into two parts, as shown in the diagram. If the rim of the bucket measures 30cm diameter, calculate the amount of water it will hold, correct to the nearest litre.
8. A piece of metal, in the shape of a square based pyramid of height 10cm and base sides of 5cm, is melted down and re-cast into spheres of diameter 3mm. How many spheres can be made?



9. Plastic plant pots are made in the shape of inverted truncated pyramids. The top is a square of 5cm sides, the bottom a square of 3cm sides and its height is 6cm.
  - a) Calculate its volume
  - b) How many can be filled from a bag containing 2 litres of compost.?

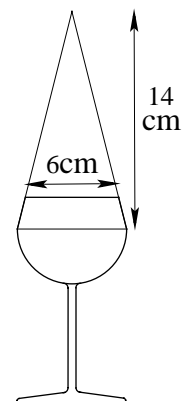


10. Tennis balls of 6cm diameter are packed into cubic boxes which hold 27 balls. Calculate
  - a) the volume of the box
  - b) the percentage of the box filled by the tennis balls.

11. A measuring cylinder with a base diameter of 3.8cm contains water. 500 spheres of diameter 5.3mm are dropped into it and are completely covered by water. Calculate the height by which the water rises.

12. A hollow metal sphere has an outside diameter of 10cm and is 1cm thick. Calculate the volume of the metal.

13. The diagram shows a wine glass. The bowl is in the shape of a hemisphere with part of a cone on top of it. The diameter of the hemisphere is 7cm and the height of the cone is 14cm. The diameter of the rim of the glass is 6cm and is 2cm above the hemisphere. Calculate the amount of wine the glass will hold, correct to the nearest millilitre.



## 52. Ratios and Scales

1. Divide £450 between Albert, Bob and Colin in the ratio 4:5:6.
2. An amount of money is divided between three people in the ratio 5:6:7. If the first gets £125, what do the others get?
3. £560 is shared among three people. The second gets twice as much as the first who receives twice as much as the third. a) Into what ratio is it divided? b) How much do they each get?
4. Mary makes 1.5 litres of lemon squash. The instructions say that the juice should be mixed with water in the ratio three parts juice to seven parts water. How much juice is needed?
5. An architect draws the plans of a house to a scale of 1:20. Complete the table below.

	Dimensions on drawing	Actual dimensions
Width of house	cm	10 metres
Height of door	11cm	m
Area of dining room floor	cm <sup>2</sup>	20m <sup>2</sup>
Angle of staircase to floor	°	40°

6. The plans of a small housing estate are drawn to a scale of 1:500. Copy and complete the following table.

	Dimensions on drawing	Actual dimensions
Length of street	cm	0.5km
Angle between two streets	°	80°
Area of school field	cm <sup>2</sup>	3000m <sup>2</sup>
Width of street	2.5cm	m

7. Mr Jones makes a doll's house for his grand-daughter. He makes it a model of his own house to a scale of 1:8. Copy and complete the following table.

	Dimensions on doll's house	Actual dimensions
Total height	cm	12m
Area of hall floor	1200cm <sup>2</sup>	m <sup>2</sup>
Volume of dining room	cm <sup>3</sup>	48m <sup>3</sup>

Mr Jones sees some model tables to put into the house. There are three designs. They are 5cm, 10cm and 15cm high. Which one should he buy?

8. A model aeroplane is made to the scale of 1:40.
  - a) If the wingspan is 20 metres, what will it be on the model?
  - b) If the wing area is 500 square centimetres on the model, what is its actual area?
  - c) The volume of the fuselage is 240 cubic metres. What is its volume on the model?

### 53. Degree of Accuracy

1. A length of wood measures 3 metres long by 10cm wide. Smaller pieces of wood are to be cut from it, each measuring 10cm by 35mm, the width correct to the nearest millimetre. Calculate the maximum and minimum number of pieces that can be cut from it.
2. Mugs are made in the shape of a cylinder, with internal dimensions of 9.3cm tall and 7.5cm diameter, both measurements correct to the nearest millimetre.
  - a) What is the maximum amount of liquid it will hold?
  - b) What is the minimum amount of liquid it will hold?
3. A garden is 15 metres long, correct to the nearest metre. A path is made with cobbles measuring 22cm long by 8.5cm wide, both dimensions correct to the nearest millimetre. The cobbles are laid side by side without any space between them, in the way shown in the

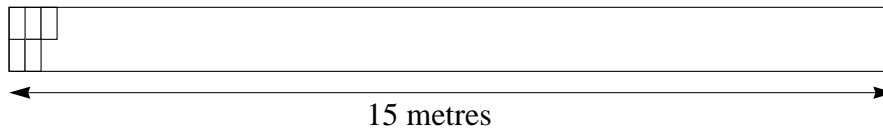
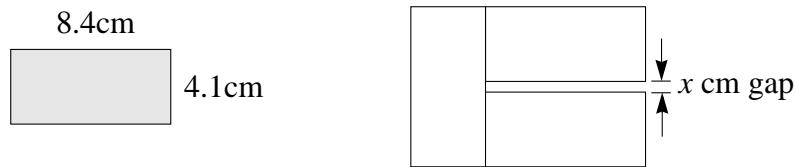


diagram with the long sides touching. How many cobbles must be ordered to ensure the path is completed?

4. Tiles are made to the dimensions shown in the diagram, correct to the nearest millimetre.



They are fitted together in the way shown with two shorter sides against a longer side, and the top and bottom edges level. Calculate the maximum and minimum values of the gap,  $x$ .

5. Erasers are made in the shape of a cuboid measuring 1cm by 2cm by 4.5cm. All dimensions correct to the nearest millimetre.
  - a) Calculate the maximum volume an eraser can be.
  - b) What is the minimum volume it can be?
  - c) If the manufacturer produces all the erasers to the lower dimensions, what is their percentage saving in rubber, over the maximum dimension?
6. The plans of a new warehouse show that the floor measures 50 metres by 26 metres, correct to the nearest metre. The concrete floor is to be covered with paint. The paint covers  $15\text{m}^2$  per litre.
  - a) What amount of paint needs to be ordered to ensure that the floor is covered?
  - b) What is the maximum amount of paint that will be left over if this is bought?
7. Floor tiles measure 20cm square, correct to the nearest millimetre. A rectangular floor measures 4.21 metres long by 3.84m wide, both measurements correct to the nearest centimetre.
  - a) What is the maximum number of tiles needed to fit on one row along the length of the floor?
  - b) What is the maximum number of tiles needed to fit along the width of the room?
  - c) What is the maximum number of tiles needed for the whole of the room?
  - d) Ian buys the maximum number of tiles needed, but finds that they have been made to their maximum size and the room measured to its minimum size. How many tiles does he have left over?

### 54. Formulae

#### Exercise 1

In each of the questions below, taking  $l$ ,  $b$ ,  $h$ ,  $r$  and  $d$  to be length, classify each of the following formulae into length, area, volume or those making no sense.

1)  $4(r^2 + lb)$

2)  $3(l + d)b$

3)  $\pi(l + b)^2$

4)  $3bd + c$

5)  $3(l \times b)$

6)  $3b(l + h)$

7)  $\frac{3(l + 2b)}{2}$

8)  $\frac{l^2 + b^2}{r}$

9)  $5\sqrt{r^2 + h^2}$

10)  $2lbh$

11)  $3lr^2$

12)  $5b^2(l + h)$

13)  $4b^2 + r$

14)  $5b(d^2 + c)$

15)  $6(l + b)$

16)  $\frac{1}{2}(l + b)$

17)  $\frac{1}{2}(bh)$

18)  $\frac{bdh}{3}$

19)  $a^2bd + e$

20)  $d\sqrt{ab} + lr$

21)  $a^2b + lrd$

#### Exercise 2

1) For the diagram shown on the right, choose one of the formulae listed below that you think best suits a) its area and b) its perimeter

(i)  $a + b + c + 1.3d$

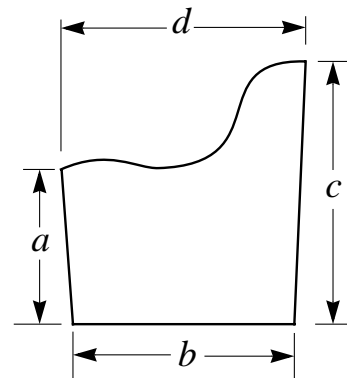
(ii)  $\frac{1}{3}a^2b$

(iii)  $\frac{b(c + a)}{2}$

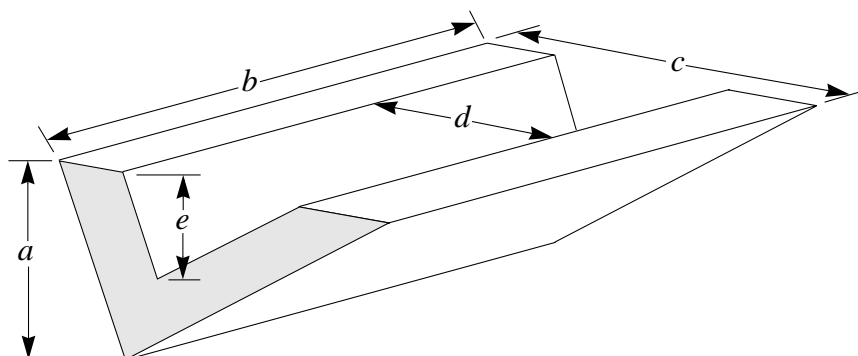
(iv)  $\sqrt{abcd}$

(v)  $\frac{1}{2}abc$

(vi)  $\frac{(a + b + c)}{2}$



2) For the diagram shown below, choose one of the formulae listed underneath you think best suits a) the area of the shaded end and b) its volume.



(i)  $\frac{1}{2}abc$

(ii)  $\frac{1}{2}(ac - de)$

(iii)  $\frac{1}{2}(ab^2c)$

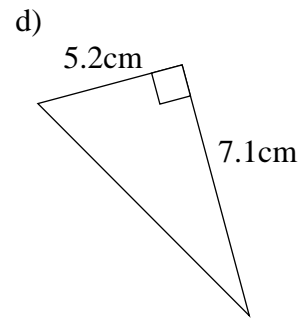
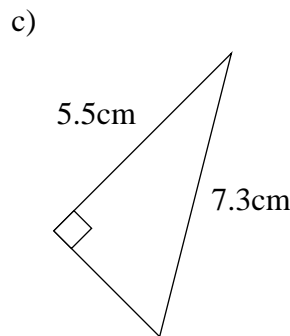
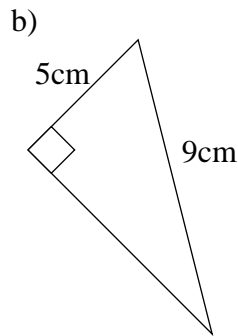
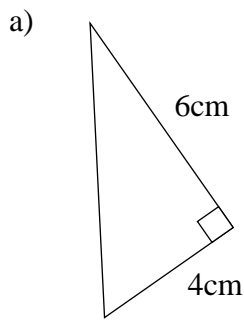
(iv)  $a^2b - bc$

(v)  $\left(\frac{ac - de}{2}\right)b$

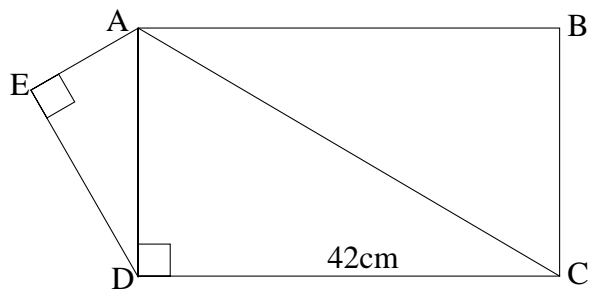
(vi)  $abc$

### 55. Pythagoras Theorem

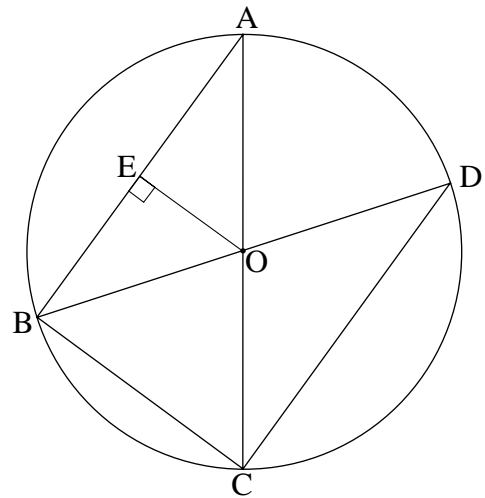
1. Calculate the length of the unknown side in each of the following diagrams



2. The diagram shows a rectangle ABCD with  $CD = 42\text{cm}$ , and the triangle ADE with  $AE = 10.5\text{cm}$  and  $DE = 14\text{cm}$ . Calculate the length of AC.



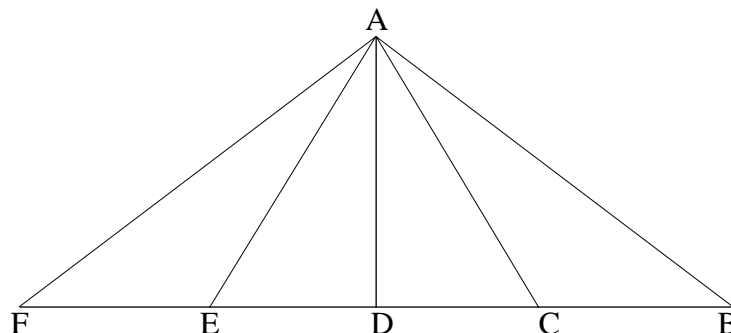
3. In the diagram on the right, AOC and BOD are diameters of a circle of radius 4.5cm. If  $DC = 7\text{cm}$ , calculate the lengths of BC and OE.



4. A rhombus has diagonals measuring 10cm and 8cm. Calculate the lengths of its sides.

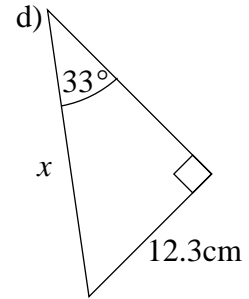
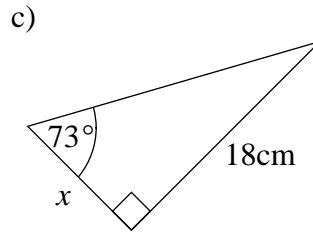
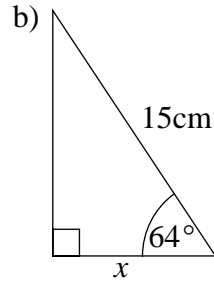
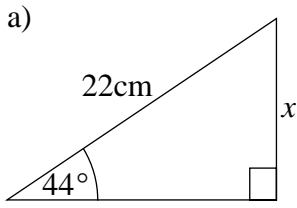
5. A window cleaner positions the base of his ladder 2 metres from the bottom of a block of flats. He opens his ladder up to 4 metres to clean the first floor window and 6.5 metres to clean the second floor window. On both occasions the ladder just touches the window sill. Calculate the distance between the two window sills.

6. A Radio mast AD is held in a vertical position by four wires, AF, AE, AC and AB.  $AF = AB$  and  $AE = AC$ . If  $AF = 50$  metres,  $AE = 35$  metres and  $AD = 25$  metres, calculate the distance CB.

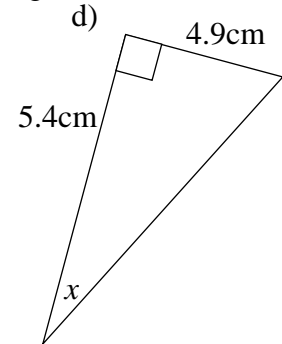
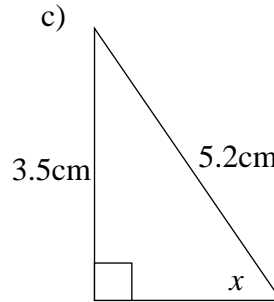
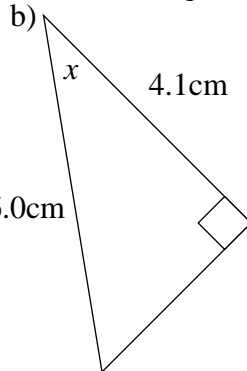
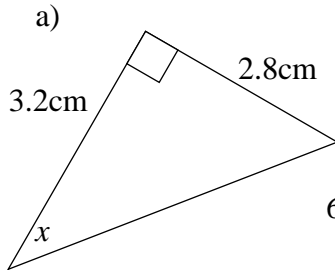


### 56. Sine, Cosine and Tangent Ratios

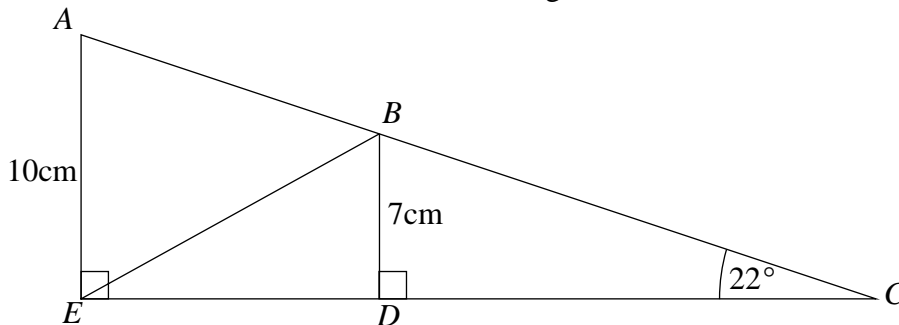
1. Calculate the length of the unknown side,  $x$ , in each of the following triangles



2. Calculate the sizes of the unknown angle,  $x$  in each of the following diagrams.

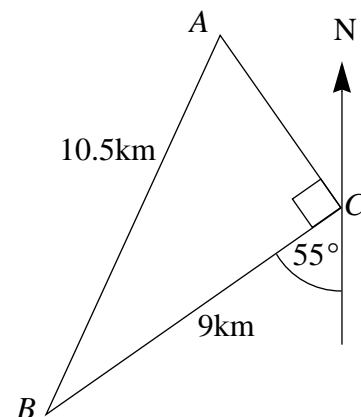


3. In the diagram below,  $AE = 10\text{cm}$ ,  $BD = 7\text{cm}$  and  $\angle ACE = 22^\circ$ . Calculate the sizes of the lines  $CD$ ,  $AC$  and  $EB$ . Calculate also the size of angle  $DEB$ .

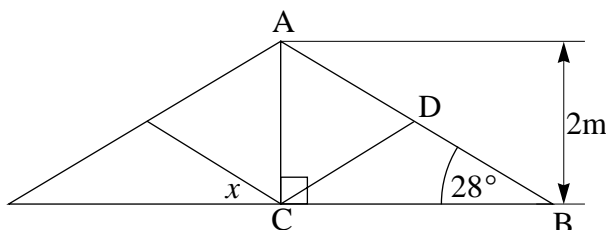


4. The diagram shows the positions of three towns,  $A$ ,  $B$  and  $C$ .

The bearing of town  $B$  from town  $C$  is  $235^\circ$ . The distance between towns  $A$  and  $B$  is  $10.5\text{km}$ , and the distance between towns  $B$  and  $C$  is  $9\text{km}$ . Calculate a) the bearing of town  $A$  from town  $B$  and b) the distance from town  $A$  to town  $C$ .

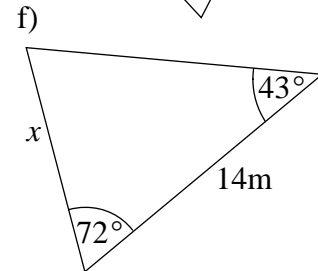
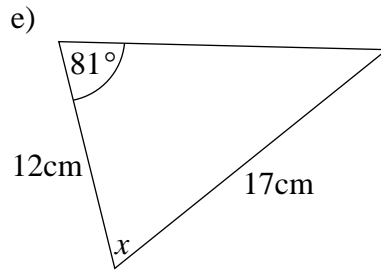
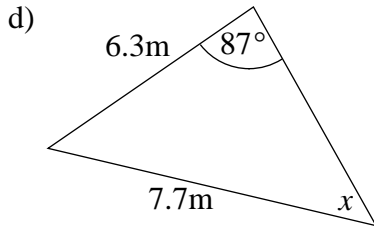
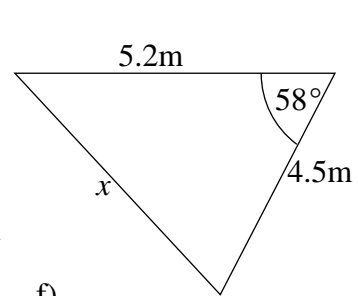
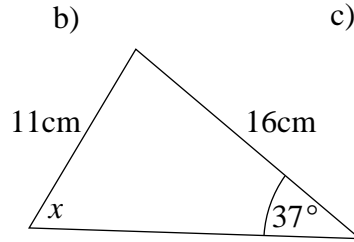
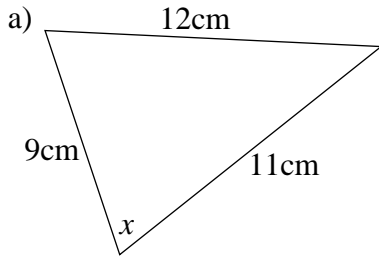


5. The diagram below shows the roof truss of a house. Its height in the middle is  $2\text{metres}$ , and the angle between the horizontal and the roof is  $28^\circ$ . It is symmetrical about the line  $AC$ . Calculate a) the width of the truss and b) the angle  $x$  if point  $D$  is half way along  $AB$ .

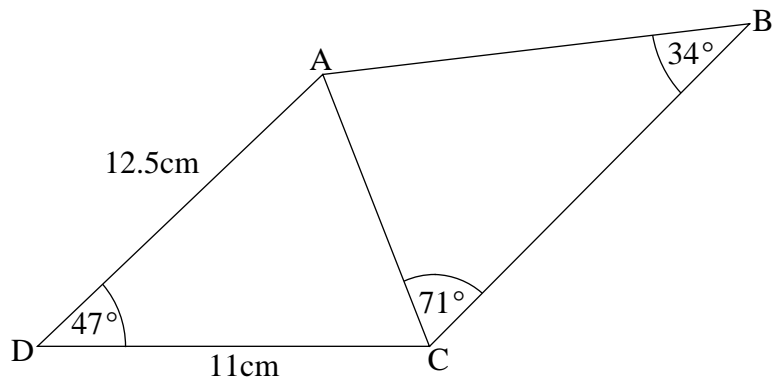


### 57. Sine and Cosine Rules

1. Calculate the sizes of the unknown value of  $x$  in each of the following triangles.



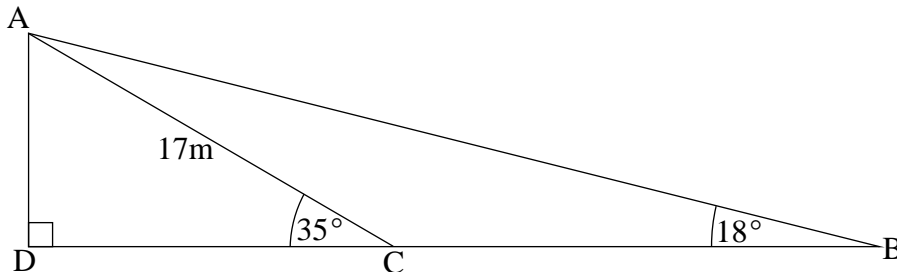
2. In the diagram on the right, calculate the length of the line AB.



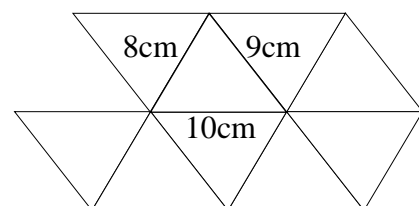
3. In a parallelogram ABCD,  $AB = DC$ , and  $AD = BC$ . The diagonal  $BD = 20\text{cm}$  and the diagonal  $AC = 14\text{cm}$ . If angle  $ACB = 85^\circ$ , calculate the sizes of the two interior angles of the parallelogram.

4. The bearing of a beacon from an aeroplane is  $160^\circ$  (S $20^\circ$  E). The aeroplane flies due south for 5km and now finds that the bearing is  $135^\circ$  (S $45^\circ$  E). What is now the distance of the beacon from the aeroplane?

5. In the diagram below, calculate the length of CB



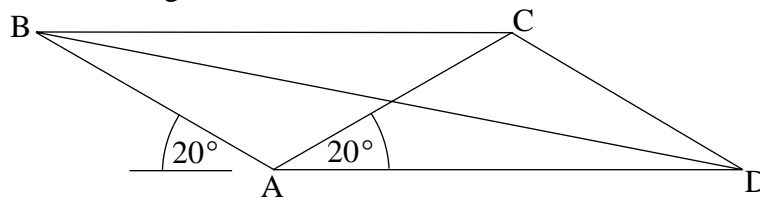
6. The diagram shows a series of congruent triangles used in the making of a patchwork design. If the sides of the triangle measure 8cm, 9cm and 10cm, calculate its angles.



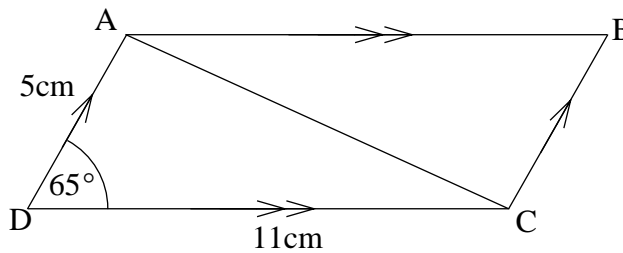
## 58. Areas of Triangles

In questions 1 to 5 calculate the areas of these triangles

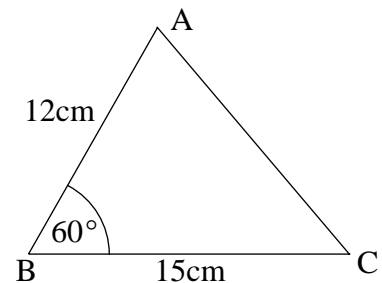
1. In triangle  $ABC$ ,  $\angle ABC = 61^\circ$ ,  $AB = 6\text{cm}$  and  $BC = 8\text{cm}$ .
2. In triangle  $XYZ$ ,  $\angle XYZ = 77^\circ$ ,  $XY = 8.5\text{cm}$  and  $YZ = 13\text{cm}$ .
3. In triangle  $CDE$ ,  $\angle CDE = 110^\circ$ ,  $CD = 5.6\text{cm}$  and  $DE = 8.5\text{cm}$ .
4. In triangle  $PQR$ ,  $\angle PQR = 119^\circ$ ,  $PQ = 7.2\text{cm}$  and  $QR = 8.7\text{cm}$ .
5. In triangle  $RST$ ,  $\angle RST = 47.7^\circ$ ,  $RS = 22.3\text{cm}$  and  $ST = 16\text{cm}$ .
6. In the diagram below both  $AB$  and  $AC$  are equal to  $9\text{cm}$ . Show that the area of triangle  $ACD$  is equal to the area of triangle  $ABD$ .



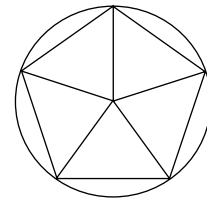
7. By calculating the area of triangle  $ADC$ , calculate the area of the parallelogram  $ABCD$ .



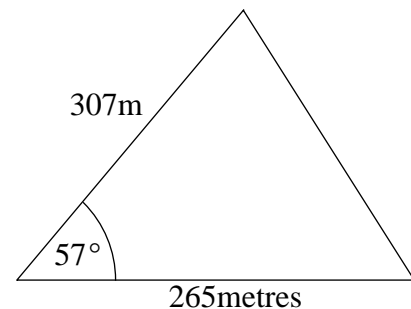
8. A patchwork bed cover is to be made from 300 pieces of fabric cut into the shape of the triangle shown. Calculate the area of the bed cover, in square metres, correct to one decimal place.



9. A regular pentagon lies inside a circle of diameter  $20\text{cm}$  with its vertices touching the circle. Calculate the area of the pentagon.



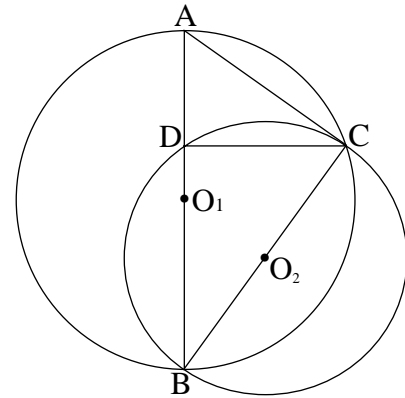
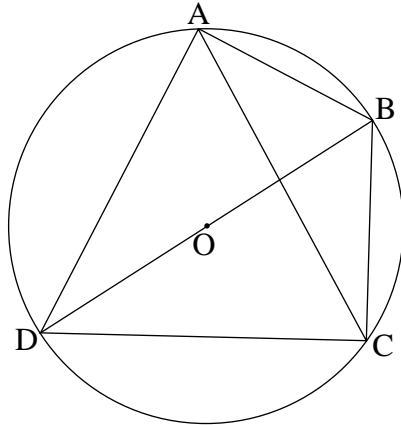
10. A field is in the shape of a triangle. Calculate its area in hectares, correct to 1 decimal place.  
(10,000 sq. metres is equal to 1 hectare)





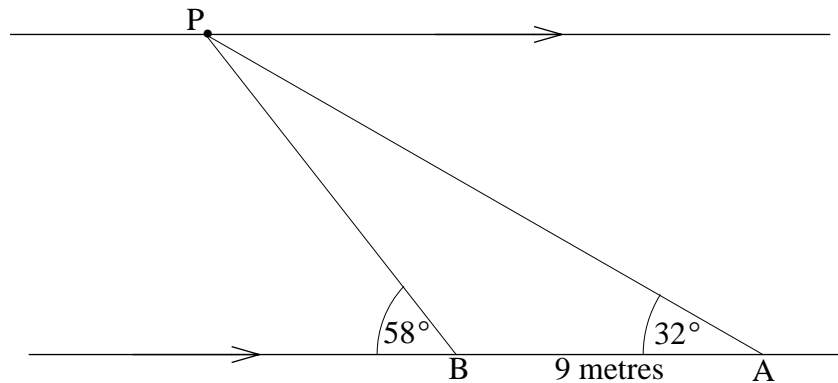
### 59. Trigonometry Mixed Exercise

- Two circles with diameters  $AO_1B$  and  $BO_2C$  intersect at B and C. If  $AB = 11\text{cm}$  and  $BC = 9\text{cm}$ , calculate the dimensions of AC and CD.
- BD, the diameter of the circle below, is 12cm in length.

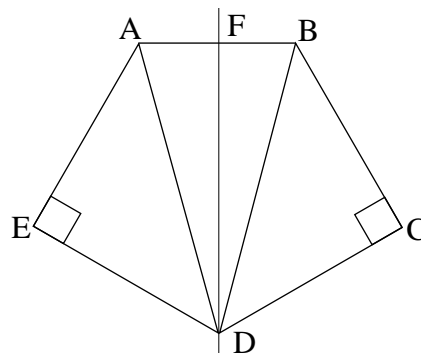


$AD = 11\text{cm}$  and  $BC = 8\text{cm}$ . Calculate the length of the line AC.

- The diagram shows a river with a post P on its bank. When viewed from two points, A and B, on the other side of the bank, the angles between the lines of sight and the bank are  $32^\circ$  and  $58^\circ$ . If the distance between A and B is 9 metres calculate the width of the river.

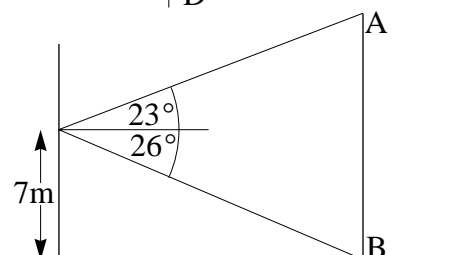


- The diagram on the right shows a pentagon ABCDE which is symmetrical about the line FD. The lengths of AE and ED are 10cm. If  $AB = 6\text{cm}$ , calculate the sizes of angles ABC and CDE.



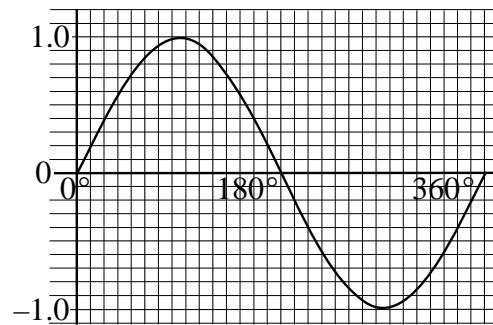
- In the triangle ABC,  $AB = AC$  and angle  $ABC = 70^\circ$ . If  $BC = 10\text{cm}$ , calculate the length of AC.

- A building AB, is observed across a street from the window of a house 7 metres above the ground. If the angle of elevation of the top of the building is  $23^\circ$  and the angle of depression of the foot of the building is  $26^\circ$ , calculate the height of the building.



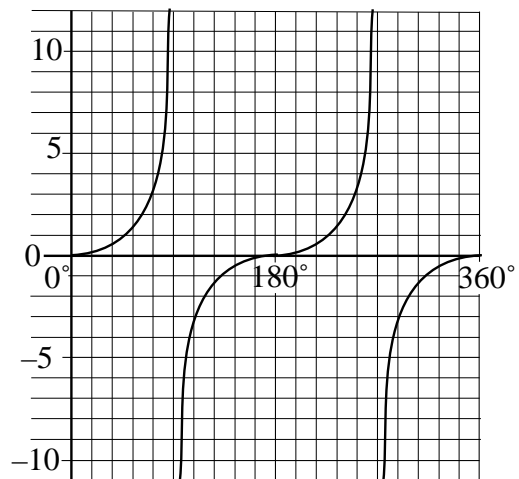
## 60. Graphs of Sines, Cosines and Tangents

1. The diagram shows the graph of  $y = \sin x$
- Sketch this into your book and mark on it the approximate solutions to the equation  $\sin x^\circ = -0.5$  where  $x$  lies between  $0^\circ$  and  $360^\circ$ .
  - Calculate accurately the solution to the equation  $\sin x^\circ = -0.5$  where  $x$  lies between  $0^\circ$  and  $360^\circ$ .



- Draw a graph of  $y = 2 \sin x^\circ + 1$  for  $0^\circ \leq x \leq 180^\circ$  using a scale of 2cm for 1 unit on the  $y$  axis and 2cm for  $30^\circ$  on the  $x$  axis.
  - From your diagram, calculate the values of  $x$  which satisfy the equation  $2 \sin x^\circ + 1 = 2.5$  for  $0^\circ \leq x \leq 180^\circ$ .
- Sketch the graph of  $y = \cos x$  for  $0^\circ \leq x \leq 360^\circ$ .
  - Show on the diagram the approximate locations of the solutions to the equation  $\cos x = -0.5$  for  $0^\circ \leq x \leq 360^\circ$ .
- Draw the graph of  $y = 3 \cos x + 2$  for  $0^\circ \leq x \leq 360^\circ$ . Use a scale of 2cm for 1 unit on the  $y$  axis and 4cm for  $90^\circ$  on the  $x$  axis.
  - From the graph, calculate the solutions to the equation  $3 \cos x^\circ + 2 = 3$  for  $0^\circ \leq x \leq 360^\circ$ .

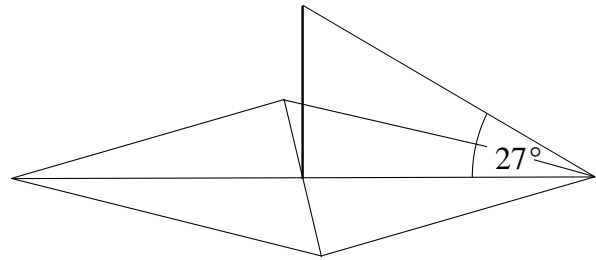
5. The diagram on the right shows a sketch of  $y = \tan x$  for  $0^\circ \leq x \leq 360^\circ$ . From the graph determine the approximate solutions to the equation  $\tan x = 4$  for  $0^\circ \leq x \leq 360^\circ$ .



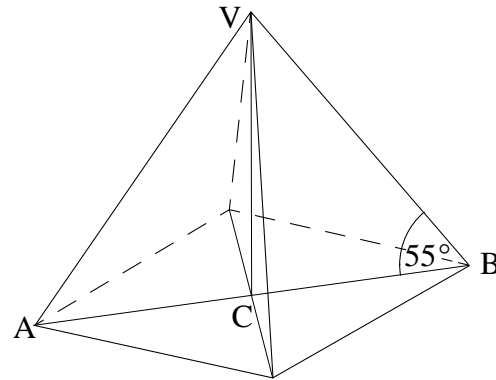
- Sketch the graph of  $y = \tan x + 2$ , for  $0^\circ \leq x \leq 360^\circ$ . Indicate on the graph the approximate location of the solution to the equation  $\tan x + 2 = 0$  for  $0^\circ \leq x \leq 360^\circ$ .
  - Sketch the graph of  $y = 3 \tan x$  on the same axis as  $y = \tan x$  for  $0^\circ \leq x \leq 180^\circ$  clearly showing the difference between the two graphs.
- Sketch the graph of  $y = 2 \tan x - 1$  for  $0^\circ \leq x \leq 360^\circ$ .
  - Indicate on the graph the approximate location of the solution to the equation  $2 \tan x - 1 = 2$  for  $0^\circ \leq x \leq 360^\circ$ .

## 61. Three Dimensional Trigonometry

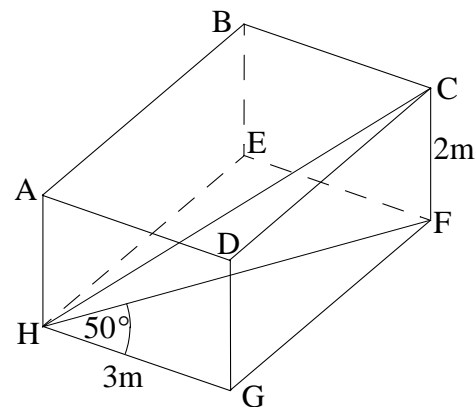
1. A flag pole stands in the middle of a square field. The angle of elevation of the top of the flag pole from one corner is  $27^\circ$ . If the flag pole is 12 metres high, calculate the lengths of the sides of the field.



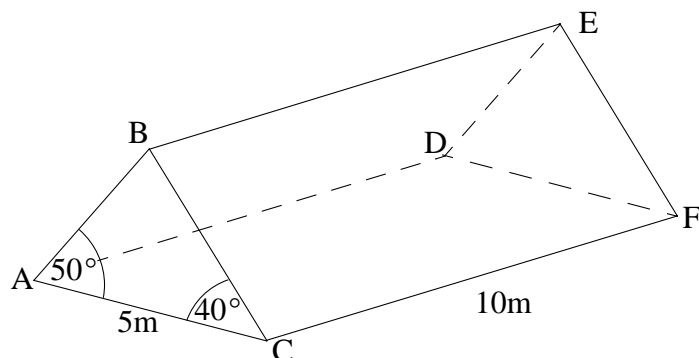
2. The diagram shows a square based right pyramid.
- If the base edges are 20cm, calculate the length of the base diagonal AB.
  - If the angle VBA is  $55^\circ$ , calculate the height VC.



3. The diagram shows a cuboid with vertices ABCDEFGH. Its height is 2 metres and its width is 3 metres. Angle GHF measures  $50^\circ$ . Calculate:
- the diagonal of the base, HF
  - angle CHF
  - the diagonal of the box HC.



4. The roof of a building is in the shape of a triangular prism and is shown in the diagram.
- Calculate:
- the length of BC
  - the angle BFC.



5. A radio mast is due east of an observer A and 150 metres away. Another observer, B, stands due north of A on level ground. If the angle of elevation of the top of the mast from B is  $28^\circ$  and the bearing of the mast from B is  $132^\circ$ , calculate the height of the mast.

## 62. Questionnaires

1. Meg is following a course in tourism. As part of the course she has to do a statistical survey. She decides to find out where most tourists in Britain come from. She thinks that the USA will be her answer.  
Her home town lies more than 300 miles from London and is very historical. She carries out a survey on the main street of her town at 12:00 midday each day during the month of July.
  - a) Will her survey be biased? Explain your answer.
  - b) How could she improve the survey?
  - c) Write down three questions she might ask.
2. Jim lives at Ayton-on-sea. He works for the local newspaper who want to begin a regular feature for people over the age of 60. To help decide whether the venture is worth undertaking, his editor asks him to do a survey of the over 60's.  
Jim decides to place a questionnaire in the newspaper and invite people to fill it in and send it back to him. This was the questionnaire.  
'The Ayton-on-sea Observer are thinking of starting a feature for the over 60's. If you are in this age range we would be grateful if you would fill in the following questionnaire and send it back to us.

Would you regularly read a feature for the over 60's? Please tick the appropriate box.

Yes

No

What types of articles would you like included? Please list them in the space below.

- a) What other information do you think is needed?
  - b) Is the survey biased in any way?
3. Jane believes that the life expectancy of a car is 15 years. She carries out a survey by observing cars travelling down her high street and noting their registration number (this tells her how old they are).
    - a) Explain why this will only give her an idea of the life expectancy and not the full answer.
    - b) In order to add to this data, she did a further survey by questioning people leaving a supermarket. What questions do you think she needs to ask?

### 63. Sampling 1

1) The table shows the number of pupils in each year at Clivedenbrook Community College.

Year Group	7	8	9	10	11	12	13
Number of Pupils	173	147	166	144	140	49	21

Emma does a survey to find out what types of TV programmes people like to watch. She decides to interview 70 pupils.

- Use a stratified sampling technique to decide how many pupils should be asked from each year.
- Without using the names of any specific TV programmes, write down three relevant questions she could ask with a choice of at least 2 replies in each case.

2) The table shows the salaries of the 180 employees at the ACME sausage factory.

Amount Earned	<£10,000	£10,000 to <£15,000	£15,000 to <£20,000	£20,000 to <£30,000	£30,000 and over
Number of Employees	45	65	44	16	10

In order to understand whether they have satisfactory working conditions the management plan to interview 40 employees. They are to be chosen at random using a stratified sampling technique.

- Calculate how many employees from each group should be interviewed.
- Write down two questions which could be asked, each with a choice of two responses.
- What other type of stratification could be used? Give one example.

3) The manager of a fitness centre wants to find out whether the members are satisfied with the amenities available. She would like ideas to improve the facilities. The table shows the age ranges for all the members.

Age Range	Under 25	25 to 40	40 to 60	Over 60
Number of Members	237	463	414	136

She decides to interview 100 members using a stratified sampling technique.

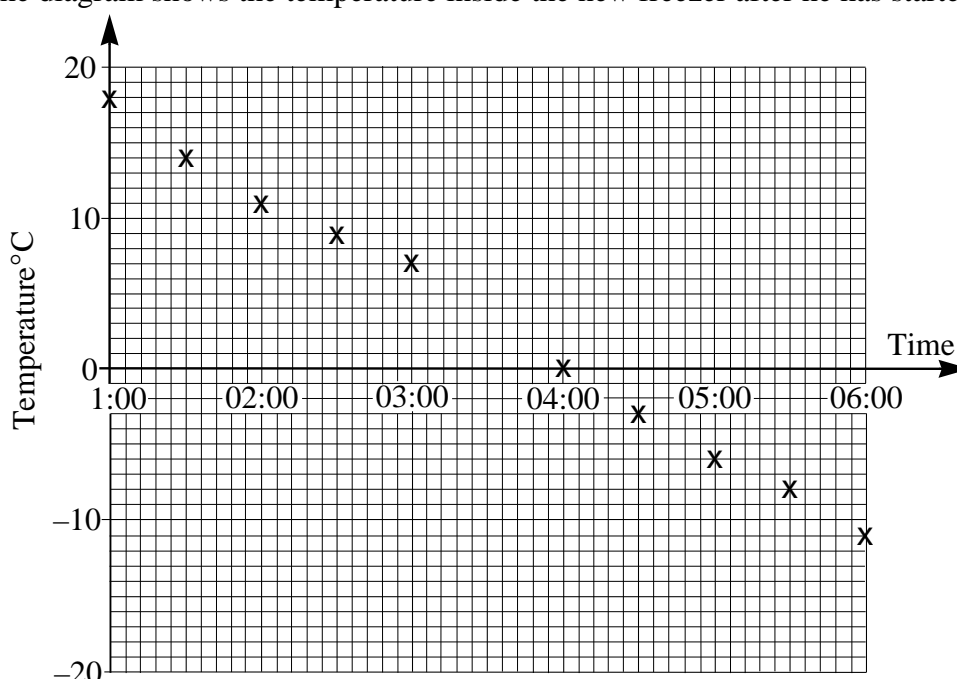
- Calculate the number of people from each group that should be interviewed.
- Write down two relevant questions, each with a choice from at least two responses.

### 64. Scatter Diagrams

1. The table shows the sales of cans of drink from a vending machine, over a period of 12 days in June

Day	S	M	T	W	T	F	S	S	M	T	W	T
Temp. °C	15	17	20	25	21	18	15	20	23	27	30	29
Cans sold	104	113	188	275	212	150	90	205	251	330	425	404

- a) Construct a scatter diagram of the data and draw on it a line of best fit.  
 b) Approximately how many cans would be sold on a day when the average temperature is 22°C?  
 2. Timothy’s old freezer registers  $-4^{\circ}\text{C}$ , which is higher than it should be so he buys a new one. The diagram shows the temperature inside the new freezer after he has started it up.



The freezer is switched on at 1:00pm and he takes the temperature every 30 minutes.

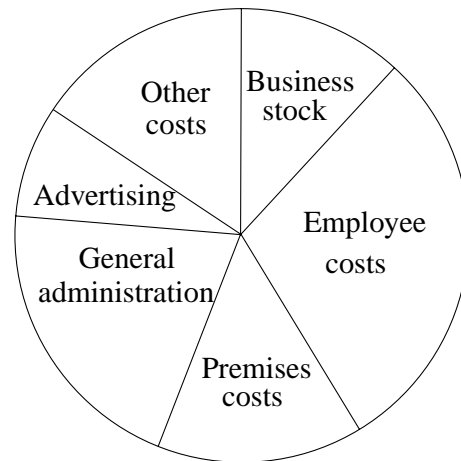
- a) At what time would you expect it to reach its minimum temperature of  $-20^{\circ}\text{C}$ ?  
 b) He missed taking the temperature at 03:30. What was the approximate temperature?  
 c) Tim transfers food from his old freezer to the new one when both the temperatures are equal. At approximately what time does he do this?  
 3. A lorry can carry up to 20 tonnes of sand. The lorry moves sand from the quarry to a collection point 2km away. The times it takes for 10 deliveries are shown in the table below. Draw a scatter graph of the data showing a line of best fit.

Time taken (minutes)	5.0	4.5	3.5	3.7	4.8	4.3	4.1	4.0	3.6	4.2
Amount of sand (tonnes)	10	14	20	18	13	15	17	16	19	17

- a) What weight of sand would take approximately 4 minutes 20 seconds to deliver?  
 b) How long would a load of 16 tonnes take to deliver?

### 65. Pie charts

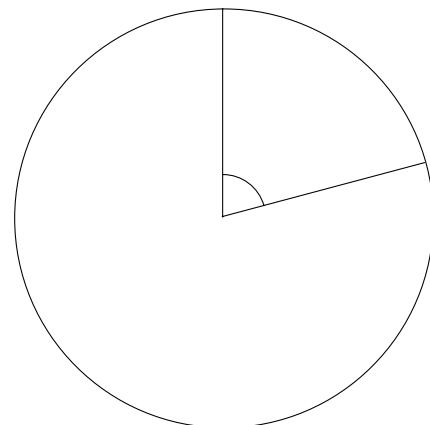
1. The diagram shows the way in which a small company spends its money.  
 During a year it spends £50,000.  
 By measuring the angles on the diagram, calculate the approximate amount of money spent in each area.



2. An Italian restaurant sells 6 different types of pizzas.  
 During one week the manager keeps records of the pizzas sold.  
 The table shows the results.  
 From the table draw a pie chart to show the data, clearly indicating how the angles are calculated.

Type of pizza	Number sold
Margherita	256
Seafood	97
Hot and spicy	83
Vegetarian	132
Chef's special	185
Hawaiian	47

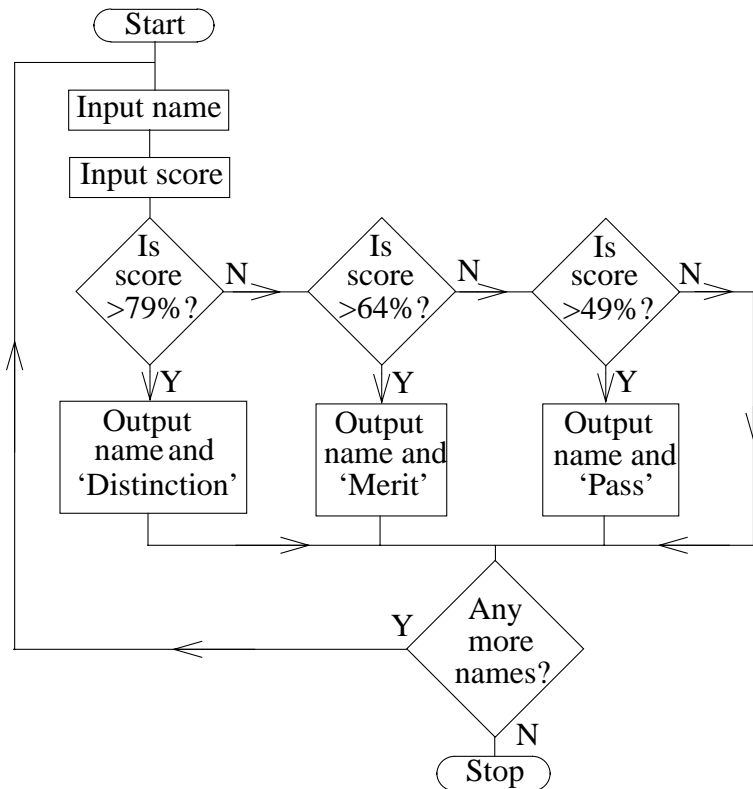
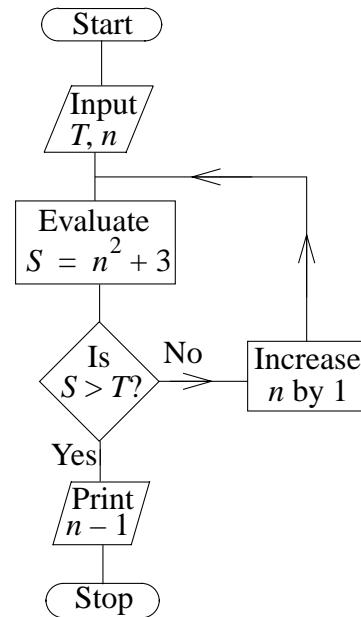
3. Jake asked a number of people how they intended to vote in the forthcoming election, for the Left, Right, Centre or Reform Party.  
 He began to draw a pie chart for the data. The angle he has drawn on this pie chart represents the 210 people who voted for the Centre Party.



- Measure the angle and calculate how many people were interviewed altogether.
- If 392 people voted for the Right Party, what angle would be used to show this?
- If  $100^\circ$  was used to represent the Left Party, show this information on a pie chart, and complete the pie chart.
- How many people intended to vote for the Reform Party?

### 66. Flowcharts 1

1. This flowchart can be used to calculate how many numbers ( $n$ ) there are in the sequence  $n^2 + 3$  smaller than or equal to  $T$ . For example, the first number is  $1^2 + 3 = 4$ , the second is  $2^2 + 3 = 7$  and so on. Starting with  $n = 40$  and  $T = 2000$ , list each successive value of  $n$  and each corresponding value of  $S$ . Find how many numbers there are in the sequence which are less than 2000



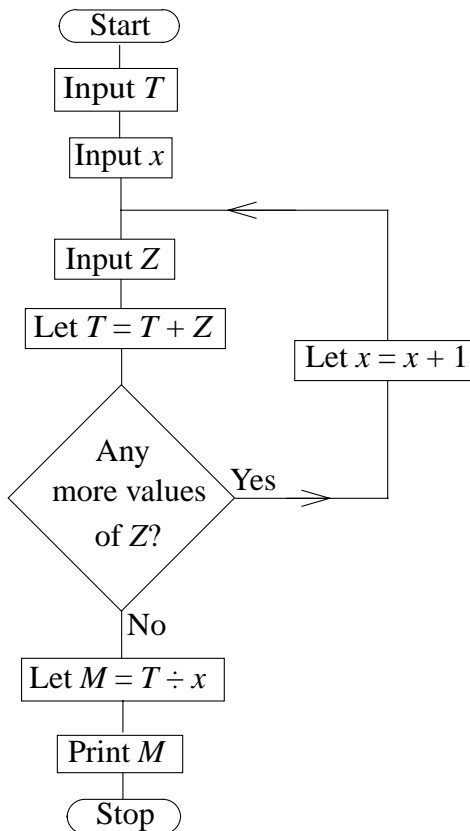
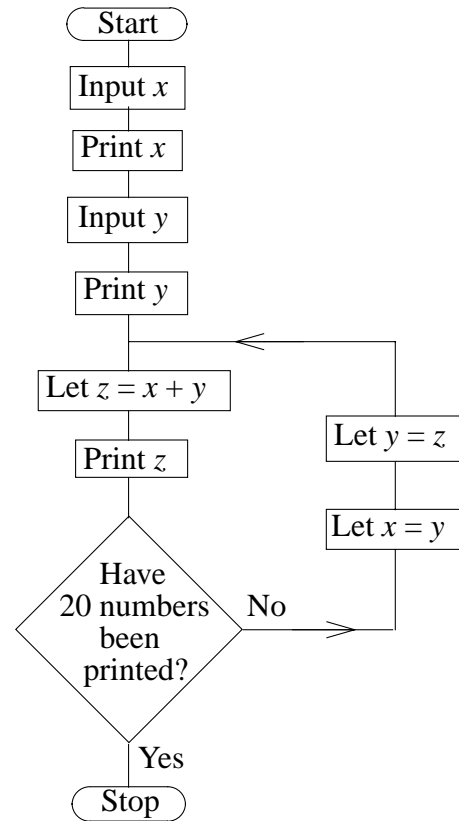
2. This flowchart can be used to print out a list of students names and their examination results. If the following are inputted, write down what you would expect to be outputted.

W Jones	67
J Connah	56
C Smith	83
R Rogers	24
H Patel	52



### 67. Flowcharts 2

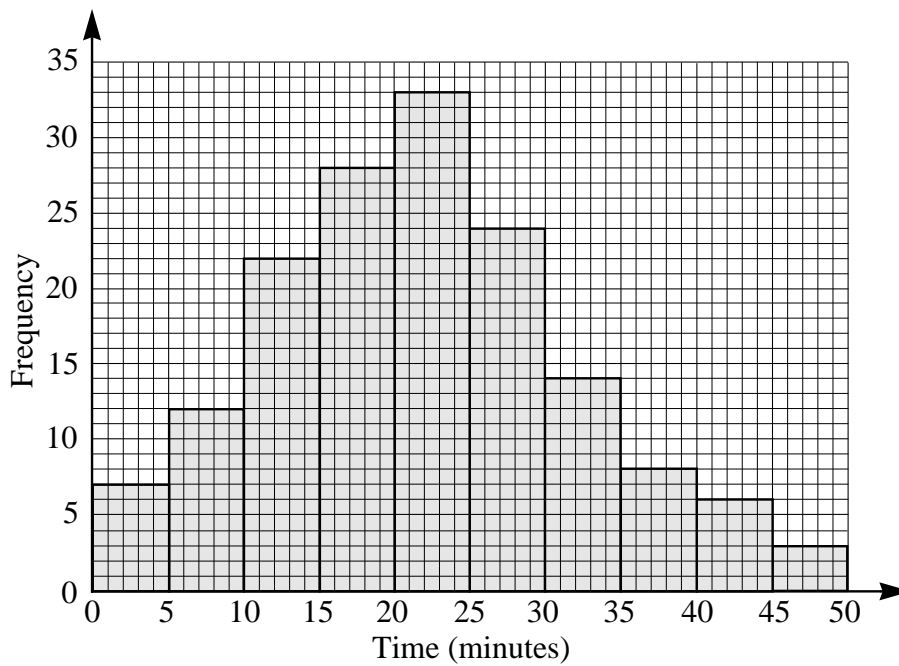
1. The flow diagram can be used to calculate the first 20 elements of a number sequence. Use it to calculate the first 20 numbers of the sequence. Explain how each number in the sequence is formed. Start with  $x = 0$  and  $y = 1$ .



2. This flow chart is used to process the following values of  $Z$ .  
 10, 7, 9, 11, 6, 3, 6, 13, 5, 10, 8.  
 Carry out the process. Write down the value of  $M$  and explain what the flowchart does. Start the process with  $T = 0$  and  $x = 1$ .

### 68. Histograms 1- Bar Charts

1. The time taken for the pupils in year 11 to get to school on Monday morning are shown below in the diagram.



a) Copy and complete this frequency chart.

Time	0-	5-	10-	15-	20-	25-	30-	35-	40-	45-50
Freq										

Use the data to calculate

- b) the mean time
- c) the percentage of pupils who took 25 minutes or more.

2. The members of a fitness centre were weighed and their masses noted in the table below.

51	53	61	77	82	93	107	67	73	70	69
51	81	99	105	47	64	77	69	82	41	65
79	62	108	98	80	75	61	65	52	41	43
69	73	78	81	73	63	74	86	42	56	58
64	81	76	63	84	92	103	94	85	72	63

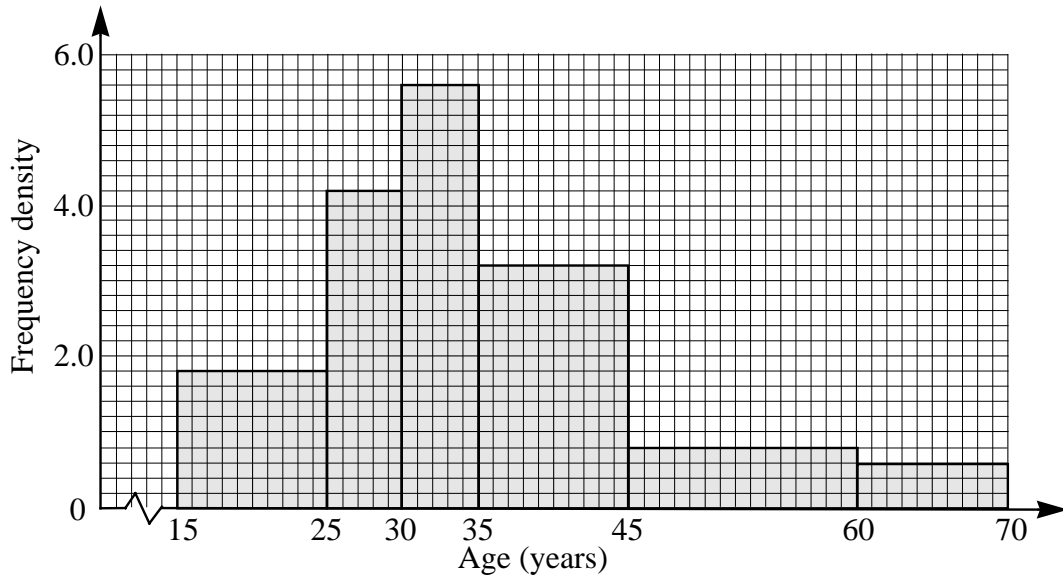
a) Copy and complete this frequency table.

Mass	40-	50-	60-	70-	80-	90-	100-110
Frequency							

- b) Show this data on a bar chart
- c) What is the modal class?

### 69. Histograms 2

1. The histogram shows the range of ages of members of a sports centre.

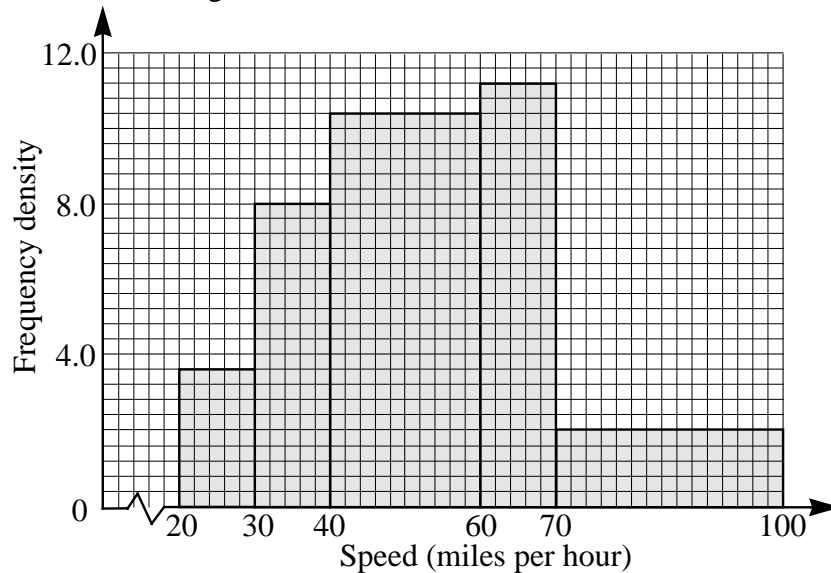


a) Complete this frequency table

Age	15-	25-	30-	35-	45-	60-70
Number of people						

b) Calculate an estimate for the mean age of the members.

2. David carries out a survey of the speeds of vehicles passing a point on a motorway. From the data he draws this histogram.



a) Complete this table

Speed	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 100$
Number of cars					

b) Which range of speeds is the modal range?

### 70. Histograms 3

1. The following list shows the heights of the tomato plants in a greenhouse.

94	124	113	103	127	106	131	132	112	118
106	117	117	123	102	114	133	117	118	101
127	109	119	93	110	126	139	108	97	113
109	119	114	128	101	129	111	121	126	110
91	122	116	99	125	116	108	125	114	107

a) Complete the following frequency table.

Height in centimetres	Number of plants	Frequency density
$90 < x \leq 100$	5	0.5
$100 < x \leq 105$		
$105 < x \leq 115$		
$115 < x \leq 120$		
$120 < x \leq 130$		
$130 < x \leq 140$		

b) Draw a histogram of the data.

c) From your histogram, find the percentage of tomato plants which are greater than 115cm.

2. A battery manufacturer tests the life of a sample of batteries by putting them into electric toys and timing them. The results she gets, in minutes, from 58 tests are shown below.

726	945	863	673	876	842	645	942	621	833
1042	526	735	893	621	773	531	733	635	998
954	763	1073	550	725	1084	747	849	716	1032
721	962	683	768	872	632	787	641	752	800
1063	794	1062	613	714	867	590	749	854	943
1021	681	943	842	841	730	961	982		

a) Complete the following frequency table

Time - Minutes	Frequency	Frequency density
$500 < x \leq 600$	4	0.04
$600 < x \leq 700$		
$700 < x \leq 750$		
$750 < x \leq 900$		
$900 < x \leq 1100$		

b) Draw a histogram to show this data

c) From the frequency table, estimate the percentage of batteries whose life expectancy is less than 800 minutes.

## 71. Mean

1. The table below shows the wages paid to a number of people working in a factory. Complete the table and calculate the mean wage.

Wages, £	Frequency	Mid value	Frequency $\times$ Mid value
$60 \leq \text{£} < 100$	4		
$100 \leq \text{£} < 140$	19		
$140 \leq \text{£} < 170$	24		
$170 \leq \text{£} < 200$	11		
$200 \leq \text{£} < 220$	6		

2. The table below shows the heights of a number of rose trees at a garden centre. Copy and complete the table of results. Calculate the approximate mean height of the roses.

Height of plant, $h$ , centimetres	Frequency		
$50 \leq h < 70$	5		
$70 \leq h < 90$	14		
$90 \leq h < 100$	16		
$100 \leq h < 110$	24		
$110 \leq h < 120$	21		
$120 \leq h < 140$	23		
$140 \leq h < 160$	17		

3. The table below shows the speeds of 60 vehicles passing a certain point on a motorway.

27.6	58.5	80.5	64.8	54.8	46.6	77.9	84.1	54.9	59.6
64.1	45.8	43.6	30.6	73.9	28.5	43.1	43.9	39.5	49.6
40.4	76.0	24.7	48.6	45.8	75.6	22.5	58.9	45.5	60.8
37.4	42.8	54.8	35.9	45.2	32.6	83.5	43.9	39.4	42.4
51.6	47.9	33.7	57.8	33.6	57.2	54.9	64.5	61.0	73.6
32.1	67.9	57.8	75.7	23.6	52.0	38.6	54.2	27.3	55.8

Make a frequency table from the values and hence calculate the approximate mean speed of the traffic, in miles per hour.

### 72. Mean, Median and Mode.

1. 30 pupils in a class were asked to keep a record of the number of pints of milk their family bought during one week. The results are given below.

18	21	15	21	22	14	14	28	21	14
14	17	21	22	14	15	21	21	16	24
15	13	14	21	25	21	14	18	12	22

- What was the modal number of pints bought per week?
- What was the mean amount of milk bought, correct to the nearest 0.1 pint?
- What was the median amount of milk bought?

2. The length of the words in the first two sentences of *Pride and Prejudice* by Jane Austin are given in the table on the right.

No. of letters	Frequency
1	6
2	21
3	9
4	10
5	9
6	2
7	2
8	5
9	1
10	2
11	2
12	1
13	1

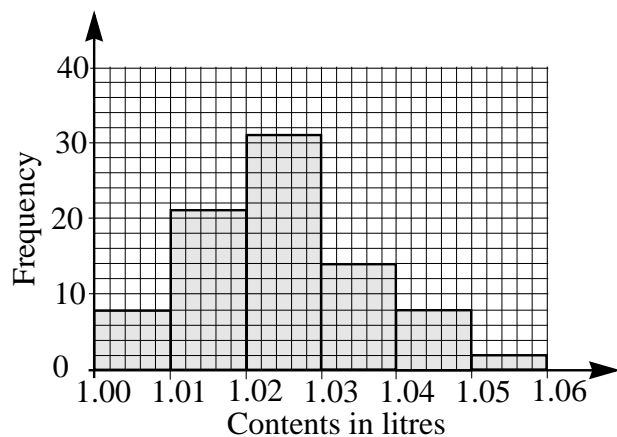
- Calculate the mean length of the words (correct to 1 decimal place)
- Calculate the median number of letters per word.
- What is the modal number of letters in a word?
- Do you think that these values are a fair indication of the words in the whole book? Explain.

3. The table on the right shows the weights of 100 packages brought into a post office during one day.

Weight $w$	Frequency	Mid Value
$60g < w \leq 100g$	21	
$100g < w \leq 300g$	44	
$300g < w \leq 600g$	17	
$600g < w \leq 1kg$	9	
$1kg < w \leq 2kg$	5	
$2kg < w \leq 3kg$	4	

- Complete the table for the mid values.
- Calculate an estimate for the mean weight of a package brought in that day.
- In which class interval does the median value lie?
- What is the modal class interval?

4. 84 packs of orange juice were sampled from a days production at a drinks factory. The contents of each pack were measured. The amounts are shown in the diagram on the right.



- From the graph:
- Write down the modal class.
  - In which class interval does the median value lie?
  - Calculate an estimate for the mean contents of a pack.

### 73. Mean and Standard Deviation

1. Nigel weighs 10 bags of potatoes. His readings are:

5.01kg, 5.03kg, 5.07kg, 5.05kg, 5.08kg, 5.04kg, 5.03kg, 5.01kg, 5.04kg, 5.08kg.

a) Calculate the mean and standard deviation of the readings.

b) Nigel notices a sign on the scales saying that all measurements are undersize by 0.02kg.

Explain the effect this has on (i) the mean (ii) the standard deviation.

2. The heights of the children in year 10 were measured by the school nurse. Her results are shown below.

Height of child (cm)	Frequency $f$	Mid point $x$
$120 < h \leq 130$	3	
$130 < h \leq 140$	7	
$140 < h \leq 150$	20	
$150 < h \leq 160$	54	
$160 < h \leq 170$	32	
$170 < h \leq 180$	9	

a) Complete the table for the mid point values.

b) Use the mid point values to calculate (i) the mean (ii) the standard deviation, for the heights of the children, giving your answer correct to one decimal place.

3. The weekly wages of the employees at a supermarket are given in the table below.

Wage £	Frequency $f$	Mid Value $x$
$80 < £ \leq 120$	5	
$120 < £ \leq 160$	8	
$160 < £ \leq 200$	9	
$200 < £ \leq 240$	11	
$240 < £ \leq 280$	14	
$280 < £ \leq 320$	17	
$320 < £ \leq 360$	3	

a) Complete the table for the mid values.

b) Using the mid point values, calculate the mean and standard deviation of the distribution, giving your answers correct to two decimal places.

c) It is decided to give each person in the store a bonus of £10 for one week. What effect will this have on the mean and the standard deviation for that week?

4. The list below shows the ages of the 30 members of a tennis club.

21	65	33	29	32	29	53	61	39	48
42	45	46	27	48	35	46	25	43	59
24	37	44	63	39	22	38	33	55	51

a) Complete this grouped frequency table.

Age (years)	Frequency	Mid value
$20 \leq a < 30$		
etc		

b) Use this table of values to calculate an estimate of the mean and standard deviation of the distribution.

c) At the Badminton club, the mean age is 48 and the standard deviation is 8.4. Comment on the differences between these figures and those for the tennis club.

### 74. Frequency Polygons

1. The data below shows the weights of 30 cats treated by a vet over a period of a week.

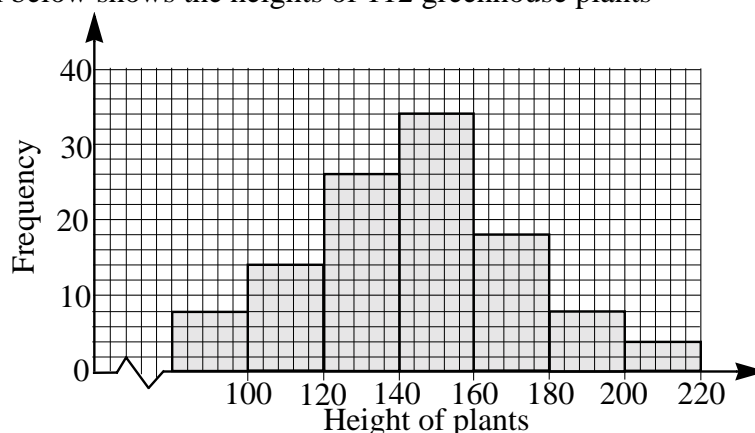
3.1    4.9    4.1    3.7    4.3    3.6    4.3    4.8    4.2    4.2  
 3.7    4.1    5.4    4.6    3.1    4.2    3.3    4.4    4.4    3.7  
 3.3    4.3    4.7    5.3    4.1    4.6    5.1    3.6    4.1    3.6

a) Use the data to complete this grouped frequency table.

Weight $w$ kg	Frequency	Mid value
$3.0 < w \leq 3.5$		
$3.5 < w \leq 4.0$		
$4.0 < w \leq 4.5$		
$4.5 < w \leq 5.0$		
$5.0 < w \leq 5.5$		

b) From your table construct a frequency polygon for the data. Use a scale of 4cm to represent 1kg on the horizontal axis and 2cm to represent 2 cats on the vertical axis.

2. The histogram below shows the heights of 112 greenhouse plants



a) Use the histogram to complete this grouped frequency table.

Height $h$ cm	Frequency	Mid value
$80 < h \leq 100$		
$100 < h \leq 120$		
$120 < h \leq 140$		
$140 < h \leq 160$		
etc.		

b) Use the frequency table to plot a frequency polygon for the data. Use a scale of 1cm to represent 10 cm on the horizontal axis and 2cm to represent 4 plants on the vertical axis.



## 75. Cumulative Frequency (1)

1. The table below shows the lengths ( $l$ ) of 100 engineering components taken at random from one days production in a light engineering works.

Length $l$	$l \leq 9.7$	$9.7 < l \leq 9.8$	$9.8 < l \leq 9.9$	$9.9 < l \leq 10.0$	$10.0 < l \leq 10.1$	$10.1 < l \leq 10.2$
Frequency	4	13	24	36	17	6

- a) Copy and complete this cumulative frequency table for this data.

Length $l$	9.7	9.8	9.9	10.0	10.1	10.2
Cumulative Frequency	4	17				

- b) Draw a cumulative frequency diagram. Use the scale of 2cm for 10 components on the vertical axis and 2cm for 0.1 unit on the horizontal axis.

From your diagram find:

- c) The median length.  
 d) The upper and lower quartiles and hence the interquartile range.  
 e) The components measuring 9.75cm or less are under size and will be scrapped. Approximately what percentage will be scrapped?

2. The table shows the ages of the 250 employees at the headquarters of 'MoneyBankPlc'.

Age	16-	20-	30-	40-	50-	60-65
Frequency	17	65	78	51	30	9

- a) Draw up a cumulative frequency table for this data.  
 b) From the table draw a cumulative frequency diagram. Use a scale of 1cm to represent 10 employees on the vertical axis and 1cm to represent 5 years on the horizontal axis.  
 c) The company decide to offer early retirement to those who are 45 or over. From the diagram estimate approximately what percentage of employees this will involve.

3. The table shows the marks gained by 200 students taking a mathematics examination.

Mark	$\leq 10$	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	3	7	22	31	34	37	28	19	13	6

- a) Draw up a cumulative frequency table from this data.  
 b) From the table draw a cumulative frequency diagram. Use the scale of 1cm to represent 10 pupils on the vertical axis and 2cm to represent 20 marks on the horizontal axis.  
 c) If 60% of the students passed the examination, what was the approximate pass mark?  
 d) Those who get a mark of 75 or more are awarded a distinction. What percentage of the students are awarded a distinction?

## 76. Cumulative Frequency (2)

1. 'Shiny' long life bulbs are guaranteed to last at least 7,500 hours before breaking down. In order to make this guarantee, 'Shiny' did trials on 300 bulbs. These are the results:

Life of bulb (hrs)	5000 to 5999	6000 to 6999	7000 to 7999	8000 to 8999	9000 to 9999	10,000 to 11,000
Frequency of breakdown	5	20	42	95	80	58

- Make a cumulative frequency table for this data.
  - From the table, draw a cumulative frequency diagram. Use a scale of 2cm to 1000 hours on the horizontal axis and 2cm to 40 bulbs on the vertical axis.
  - From the graph, estimate the percentage of bulbs that fulfil the guarantee.
2. A flying school gives 1 hour lessons. Some of that time is spent on the ground having instruction and some in the air. The time spent in the air for 100 lessons were as follows.

Time spent in the air	20-30 mins	30-35 mins	35-40 mins	40-45 mins	45-50 mins	50-55 mins
Frequency	6	10	12	34	23	15

- Make a cumulative frequency table for this data. From the table, draw a cumulative frequency diagram. Use a scale of 1cm to 2 mins horizontally and 1cm to 5 people vertically.
  - From the diagram, estimate the median and interquartile range of the data.
3. The times that trains arrive at a station are recorded by the station staff. They record whether a train is early or late and by how much. The table below shows their data for one day.

Timing	10-5 mins early	5-0 mins early	0-5 mins late	5-10 mins late	10-15 mins late	15-20mins late
Frequency	33	66	22	17	10	2

- Make a cumulative frequency table for this data.
  - From the table, draw a cumulative frequency diagram. Use a scale of 2cm to represent 5 minutes on the horizontal axis and 1cm to represent 10 trains on the vertical axis.
  - The train company guarantees that its services will not be more than 7 minutes late. Use your graph to check what percentage were outside the guarantee.
4. A large electrical retailer keeps stocks of television sets in its main warehouse for delivery to its shops. Over a period of 1 year (365 days) the number of TV's in the warehouse are given in the table below.

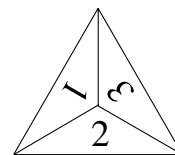
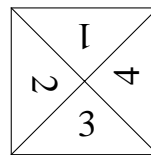
No. of TV's in warehouse	50-100	101-150	151-200	201-250	251-300	301-350
No of days (frequency)	5	27	77	145	83	28

- Make a cumulative frequency table for this data.
- Draw a cumulative frequency diagram using a scale of 2cm to represent 40 TV's on the horizontal axis, and 2cm to represent 40 days on the vertical axis.  
It is the policy of the company to keep a stock of between 130 and 260 TV's.
- For approximately how many days was the stock within these limits?
- A stock of 110 TV's is regarded as the minimum quantity that the warehouse should have. For approximately how many days was the stock below the minimum requirement?

## 77. Probability 1

1. A bag contains 4 red discs and 5 green discs. A disc is selected at random from the bag, its colour noted and then replaced. This is carried out 3 times. Calculate the probability of getting:
- a green disc on the first draw
  - a green disc followed by a red disc
  - three green discs
  - a green disc and two red discs in any order.

2. A game is played using a four sided spinner and a three sided spinner. The two spinners are spun together and their two values added together in the way shown in the diagram.

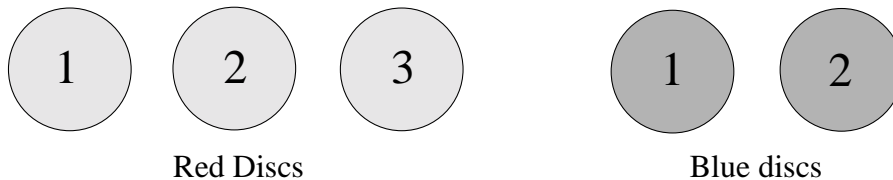


$$3+2 = 5$$

- Calculate the probability of getting an outcome of a) 3 b) 5 c) not 6
3. The probability that a parcel will be delivered the next day is 0.4. Two parcels are sent, independently of each other, on the same day. What is the probability that:
- both will be delivered the next day?
  - only one will be delivered the next day?
  - neither will be delivered the next day?
4. A biased dice has the numbers 1 to 6 on its faces. The probability of throwing a 1 is 0.1 and the probability of throwing a 2 is also 0.1. The probability of throwing each of the remaining 4 numbers is 0.2. Calculate the probability of throwing:
- a 1 followed by a 3
  - a 2 followed by a 4
  - two 2's.
5. The probability that the post arrives before 8.00 am is 0.2 and the probability that the daily newspaper arrives before 8.00 am is 0.7. Calculate the probability that:
- both newspaper and post arrive before 8.00
  - the post arrives before 8.00 but the newspaper arrives after 8.00
  - both arrive after 8.00.
  - one or both of them don't arrive before 8.00.
6. A fair dice having the numbers 1 to 6 on it is rolled and its value noted. It is rolled a second time and its value added to the first one. For example, if 3 is rolled the first time and 2 the second time, the total after 2 rolls will be 5.
- What is the probability that the score will be greater than 12?
  - What is the probability of scoring between 6 and 9 inclusive with two throws of the dice?

## 78. Probability 2

1. A bag contains 3 black discs and 7 white discs. A disc is taken at random from the bag and not replaced. This is carried out three times. Calculate the probability of getting:
  - a) a black disc on the first draw
  - b) a black disc followed by a white disc
  - c) a black disc followed by two white discs.
2. It is known from past experience that of every 350 students entered for a typing examination 280 will pass the first time. Of those that fail, 80% of them pass on their second attempt. What is the probability that a student, chosen at random from a group of students sitting the examination for the first time, will pass on the first or second attempt?
3. The probability that there will be passengers waiting to be picked up at a bus stop is 0.85. A long distance bus has 3 stops to make. Calculate the probability that it will have to pick up passengers one or more times on its journey.
4. In a game of chance, two people each have a dice numbered 1, 1, 1, 2, 2, 2. They both throw the dice together. If both the outcomes are the same, then they have a second try. If they are the same again, then they have a third try, and so on until someone gets a 2 while the other gets a 1. The winner is the person who gets the 2.
  - a) What is the probability that the game is over on the first pair of throws?
  - b) What is the probability that someone wins on the second pair of throws?
5. In a class of 30 pupils there are 14 boys and 16 girls. Two names are chosen at random. Calculate the probability that:
  - a) the first name selected is a girls'
  - b) both the names selected are girls'.
6. In a bag there are 5 discs, 3 red and 2 blue. The red discs have the numbers 1, 2 and 3 written on them and the blue discs have the numbers 1 and 2 on them.



- a) What is the probability of withdrawing a disc with a 2 on it from the bag?
  - b) Two discs are drawn from the bag. What is the probability that one disc has a 2 on it and one disc is red?
7. An engineering works produces metal rods. The rods are 6.2cm long and 0.5mm in diameter, both dimensions to the nearest 0.1mm. When the rods are inspected there can be four different outcomes:
    - either one or both dimensions are under size, in which case the rod is scrapped,
    - or one dimension is oversize and one is correct, in which case it is reworked,
    - or both dimensions are oversize, in which case the rod is reworked,
    - or both dimensions are correct, in which case the rod is acceptable.
 It is known from past experience that 1.2% are under size in length and 1.5% are under size in diameter. Also 2.5% are oversize in length and 2.2% are oversize in diameter. A rod is selected at random. What is the probability that it is:
    - a) scrapped
    - b) reworked
    - c) accepted?

## 79. Relative Probability

1. A biased coin is tossed 50 times. The number of times it comes down heads is 31 and the number of times it comes down tails is 19. If it is now tossed 1000 times, how many times would you expect it to show heads?
2. In an opinion poll for the general election, 500 people in a constituency were asked what party they would vote for. 220 said they would vote for the Left party, 170 for the Right party and 110 for the Centre party. The 500 people can be taken as representative of the population of the constituency.
  - a) What is the probability that a person chosen at random will vote for:
    - (i) the Left party      (ii) the Right party      (iii) the Centre party?
  - b) It is expected that 45,000 people will vote in the election. Estimate the number of people who vote for each party.
3. A chocolate company make sugar coated chocolate sweets in red, yellow and green. It is known that more people prefer red sweets than any other colour so the company mix them in the ratio 5 red to 3 yellow to 2 green.
  - a) In a bag of 50 sweets, how many of each colour would you expect?
  - b) David takes a sweet from the bag without looking at it. What is the probability that it will be:
    - (i) red                      (ii) yellow                      (iii) green?

Sarah only likes the red sweets. She eats them all from her bag without eating any of the others. She gives the bag to David who now takes a sweet. What is the probability that it is:

    - (i) green                      (ii) yellow?

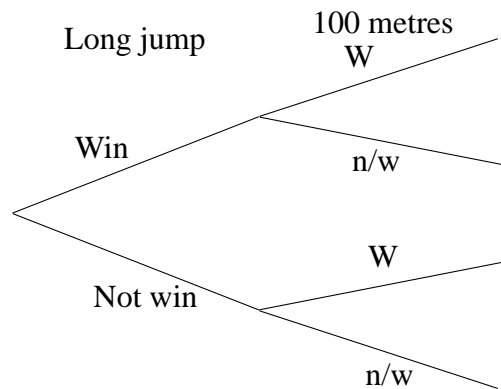
During one days production of sweets at the factory, the company make 100,000 green sweets. How many red and yellow sweets do they make?
4. An unfair triangular spinner has the numbers 1, 2 and 3 on it. It is spun 100 times. Number 1 is obtained 25 times, number 2 occurs 30 times and number 3 occurs 45 times.
  - a) Write down the probability of each number occurring.
  - b) If it is spun 550 times, how often would you expect each number to occur?
5. A survey is carried out over a period of six days to determine whether the voters are for or against having a new by-pass outside their town. Each day 50 people were asked their opinion. The results were put into the following table.

Number of people	50	100	150	200	250	300
Number 'for'	30	45	55	83	96	122
Number 'against'	20	55				
Probability 'for'	0.60	0.45				
Probability 'against'	0.40	0.55				

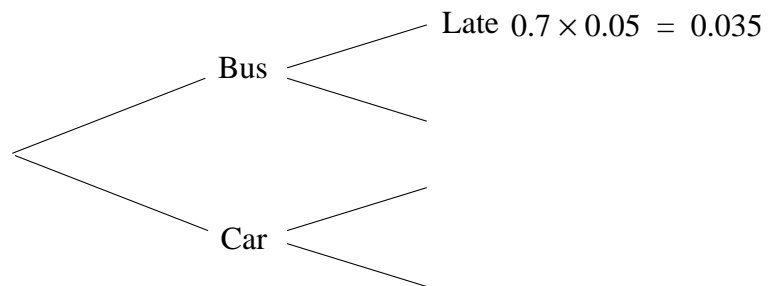
- a) Copy and complete the table.
- b) On graph paper plot 'Number of people' against 'Probability for'. Use a scale of 2cm to represent 40 people on the horizontal axis and 2cm to represent 0.1 on the vertical axis.
- c) From your graph estimate the probability that a voter chosen at random would be for the by-pass.
- d) There are 70,000 voters in the town. If the people asked were a representative sample of the voting population, how many would you expect to be against the by-pass?

### 80. Tree Diagrams

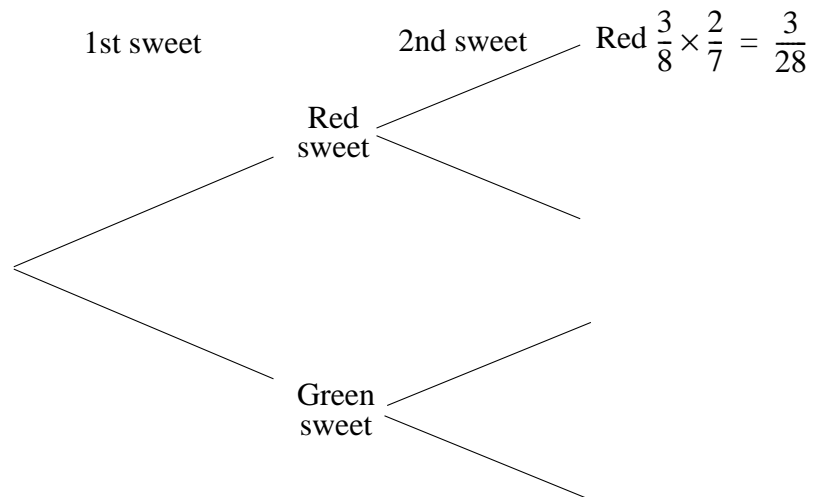
- The probability that Sarah will win the long jump is 0.1 and, independently, the probability that she will win the 100 metres is 0.15.
  - Complete the tree diagram to show all the possible outcomes.
  - What is the probability that she wins one or both events?
  - What is the probability that she wins neither event?



- William waits at the bus stop each morning in order to catch the bus to work. Some mornings his friend arrives in his car and gives him a ride to work. From past experience the probabilities of William getting the bus is 0.7 and the probability that he gets a ride with his friend is 0.3. When he gets the bus, the chance of him being late for work is 0.05 and when he gets a ride the chance of him being late is 0.15.
  - Complete the tree diagram to show all the possible outcomes.
  - From the diagram calculate the probability that on a day chosen at random he will be late.



- A bag contains 5 green sweets and 3 red sweets. Two sweets are withdrawn at random from the bag.
  - Complete the diagram to show all the possible outcomes
  - What is the probability of getting:
    - a sweet of each colour
    - at least one red sweet?

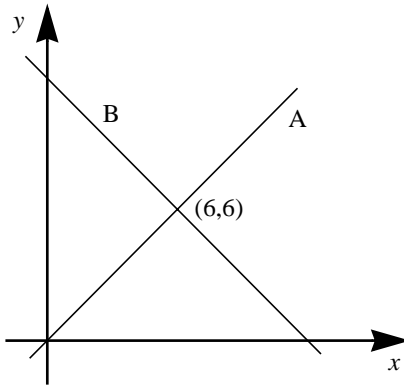


- A bag contains 8 discs, 4 black and 4 white. 2 discs are removed at random from the bag. By drawing a tree diagram calculate the probability that:
  - both discs will be black
  - at least one of the discs will be white.
- In class 7B, two pupils are chosen at random from the class register to be the monitors. There are 12 boys and 16 girls in the class. By drawing a tree diagram calculate the probability that:
  - the two people chosen will be girls
  - the two people chosen will be boys
  - one person of each gender will be chosen.

## 81 Perpendicular Lines

- 1) a) What is the gradient of a line parallel to  $y = 3x$ ?  
 b) What is the gradient of a line perpendicular to  $y = 3x$ ?  
 c) Which of the following equations represent lines perpendicular to the line  $y = 3x$ ?  
 (i)  $y = 4 - \frac{x}{3}$       (ii)  $y - 3x = 2$       (iii)  $y = -\frac{x}{3} + 7$       (iv)  $y + \frac{x}{3} = 6$   
 (v)  $y = -3x - 1$       (vi)  $3y - 9x = 7$       (vii)  $3y + x = 9$

2)



The diagram shows two straight lines A and B.

Line A goes through points (0,0) and (6,6).

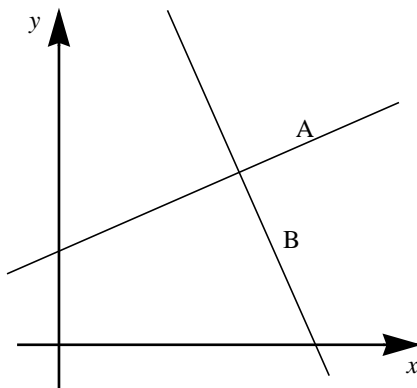
a) What is the equation of line A in the form  $y = mx + c$ ?

Line B is perpendicular to line A.

Lines A and B meet at (6,6).

b) What is the equation of line B?

3)



The diagram shows two straight lines A and B.

Line A cuts the y axis at (0,2) and meets line B at point (4,4).

a) What is the equation of line A in the form  $y = mx + c$ ?

Line B is perpendicular to line A.

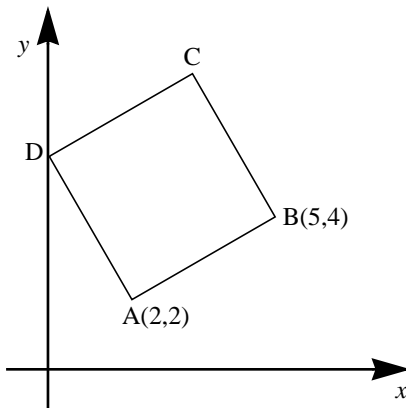
b) What is the gradient of line B?

c) At what point will line B cut the y axis?

d) What is the equation of line B?

- 4) The line  $y = \frac{x}{3} + 4$  cuts the line  $y = mx + c$  at the point (6,2). If the lines are perpendicular to each other, what is the equation of the second line?

5)



The diagram shows a square ABCD with corners A(2,2) and B(5,4).

a) What is the gradient of the line AB?

b) What is the equation of the line AB in the form  $y = mx + c$ ?

c) Write down the gradients of the other three sides.

d) What are the equations of the other three sides?

## 82 Completing the Square

1) In each of the following expressions determine the number which must be added (or subtracted) to make a perfect square. Some have been done for you.

a)  $x^2 + 6x$

b)  $x^2 + 4x$

c)  $x^2 - 3x$

$$x^2 + 6x + c^2 = (x + c)^2$$

$$x^2 + 6x + c^2 = x^2 + 2cx + c^2$$

So  $6x = 2cx$

Therefore  $c = 3$  and  $c^2 = 9$

i.e. 9 must be added to make a perfect square

d)  $x^2 - 7x$

e)  $x^2 - x$

f)  $x^2 + 9x$

g)  $3x^2 + 9x$

h)  $4x^2 - 3x$

i)  $7x^2 + 4x$

$$3x^2 + 9x + c^2 = (\sqrt{3}x + c)^2$$

$$3x^2 + 9x + c^2 = 3x^2 + 2\sqrt{3}cx + c^2$$

So  $9x = 2\sqrt{3}cx$

Therefore  $c = \frac{9}{2\sqrt{3}}$  and  $c^2 = \frac{81}{12} = \frac{27}{4}$

i.e.  $\frac{27}{4}$  must be added to make a perfect square

j)  $6x^2 + 2x$

k)  $9x^2 - 3x$

l)  $16x^2 + 5x$

2) Find the solution to these equations by first completing the square.

a)  $x^2 - 8x - 20 = 0$

b)  $x^2 - 4x - 21 = 0$

c)  $x^2 + 11x + 18 = 0$

d)  $x^2 + 5x + 6 = 0$

e)  $x^2 - x - 2 = 0$

f)  $x^2 + 3x - 4 = 0$

g)  $x^2 - 4x - 5 = 0$

h)  $x^2 + 3x - 28 = 0$

i)  $x^2 + 4x + 3 = 0$

j)  $2x^2 + 5x + 2 = 0$

k)  $3x^2 - 4x - 4 = 0$

l)  $2x^2 + 5x - 3 = 0$

m)  $8x^2 + 2x - 1 = 0$

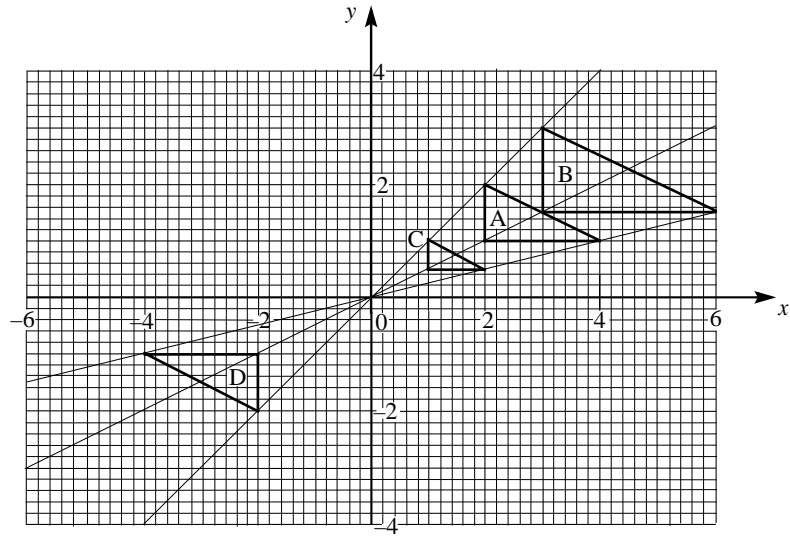
n)  $6x^2 + x - 2 = 0$

o)  $15x^2 + 4x - 3 = 0$

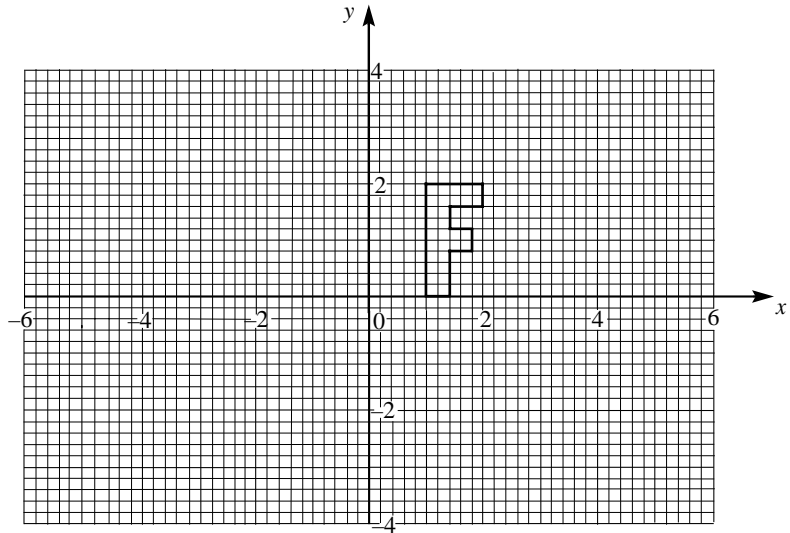


### 83 Enlargement with a Negative Scale Factor 1

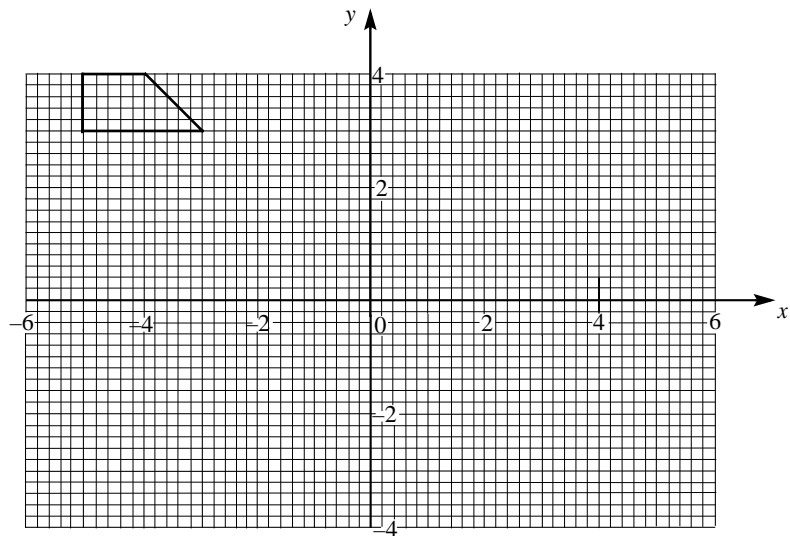
- 1) Shape A is enlarged three times to make shapes B, C and D. What are their enlargement factors?



- 2) Enlarge this letter F with a scale factor of  $-2$  through the origin

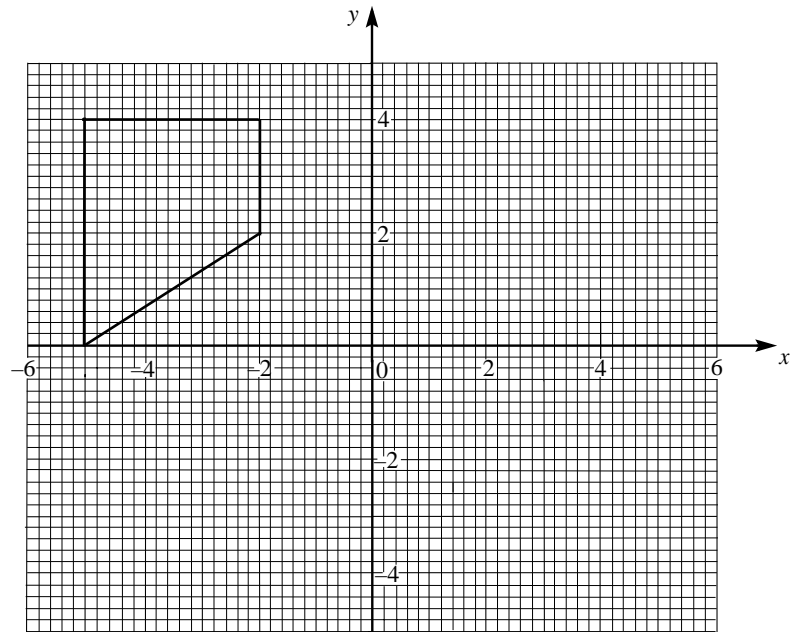


- 3) Enlarge this shape with a scale factor of  $-3$  through the point  $(-3, 2)$

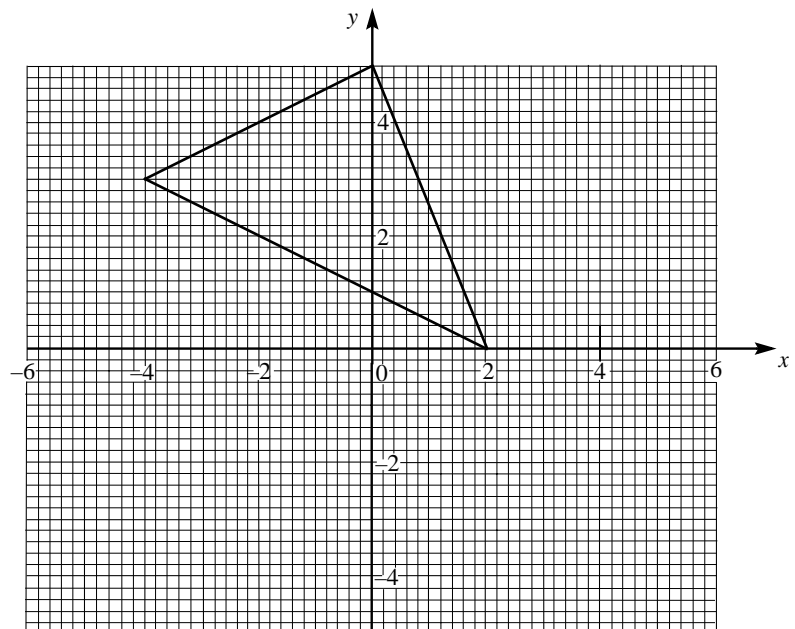


**84 Enlargement with a Negative Scale Factor 2**

- 1) Enlarge this shape with a scale factor of  $-\frac{1}{2}$  through the origin.

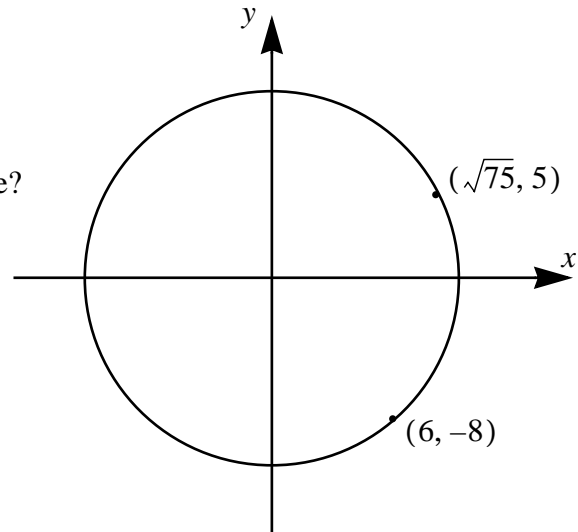


- 2) Enlarge this triangle with a scale factor of  $-\frac{1}{2}$  through the point  $(-2, 0)$



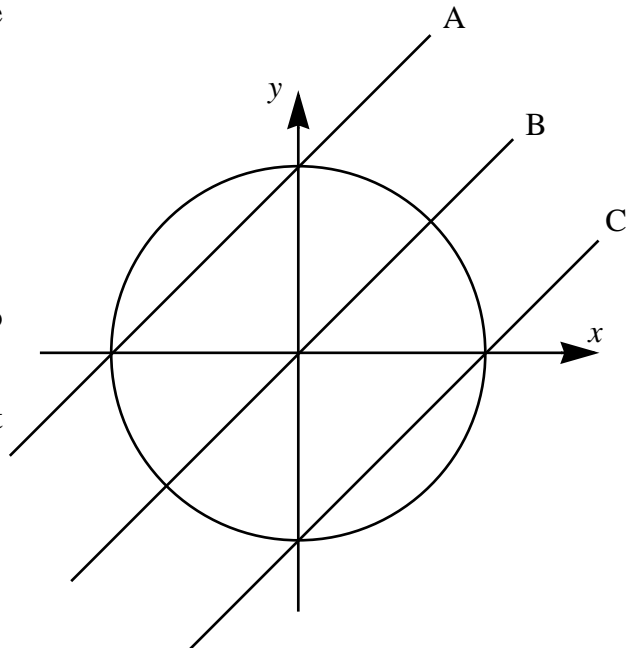
### 85 Equation of a Circle

- 1) Two points  $(\sqrt{75}, 5)$  and  $(6, -8)$  lie on a circle  $x^2 + y^2 = r^2$  where  $r$  is the radius of the circle with its centre at the origin  $(0,0)$ , What is the radius of the circle?



- 2) The diagram shows the graph of the circle  $x^2 + y^2 = r^2$  and three parallel straight lines.

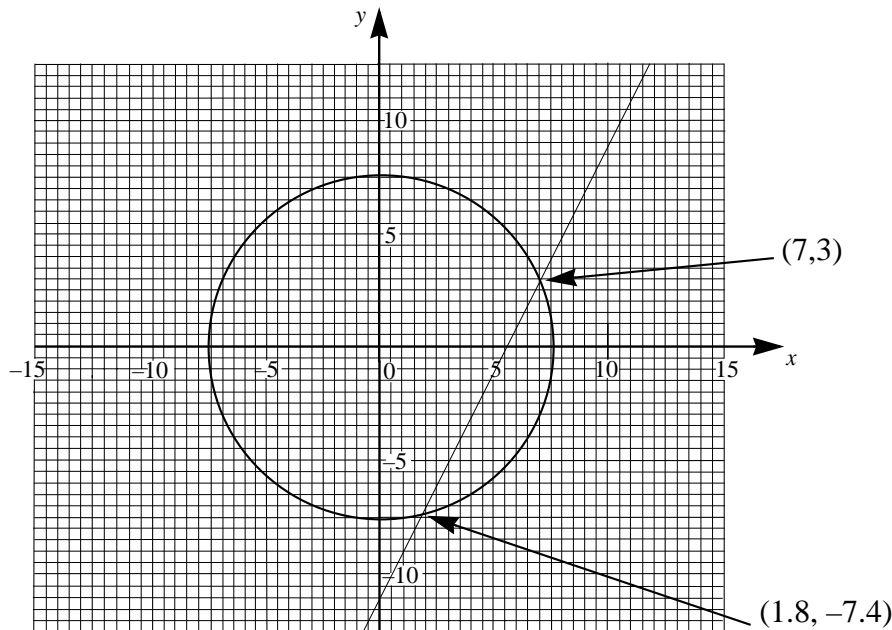
- Which line has the equation  $y = x + 6$ ?
- What is the value of  $r$ ?
- What are the equations of the other two graphs?
- At which points does graph B intersect with the circle?



- 3) At which points does the line  $y = 2x - 4$  intersect with the circle  $x^2 + y^2 = 100$ ?
- 4) a) At which points does the line  $y = -7$  cut the circle  $x^2 + y^2 = 100$ ?
- b) Write down the co-ordinates of the points at which the line  $y = 7$  cuts this circle.
- 5) One of the points at which the line  $y = \frac{1}{2}x - 2$  intersects with the circle  $x^2 + y^2 = r^2$  is  $(-5, -4.5)$
- What is the value of  $r$ ? Leave your answer as a surd.
  - What are the co-ordinates of the other point of intersection?

## 86 Simultaneous Equations 2

- 1) The points of intersection of the the straight line  $2x - y = 11$  and the circle  $x^2 + y^2 = 58$  can be found either graphically, like this:



or by solving the simultaneous equations

$$x^2 + y^2 = 58 \quad (\text{i})$$

and  $2x - y = 11 \quad (\text{ii})$

like this:

Rearrange  $2x - y = 11$  to give  $y = 2x - 11 \quad (\text{iii})$

substitute (iii) into equation (i) for y

$$x^2 + (2x - 11)^2 = 58$$

$$x^2 + 4x^2 - 44x + 121 = 58$$

$$5x^2 - 44x + 63 = 0$$

$$(5x - 9)(x - 7) = 0$$

$$x = \frac{9}{5} \quad \text{or} \quad x = 7$$

Substitute into (iii) to find y which gives

$$y = -\frac{37}{5} \quad \text{and} \quad y = 3$$

so the points of intersection of the two graphs are  $(\frac{9}{5}, -\frac{37}{5})$  and  $(7, 3)$

### Exercise

Find graphically the points of intersection of the following circles and straight lines and check the answers by solving the two simultaneous equations.

- 1)  $x^2 + y^2 = 13$     2)  $x^2 + y^2 = 41$     3)  $x^2 + y^2 = 29$     4)  $x^2 + y^2 = 61$     5)  $x^2 + y^2 = 25$   
 $y = 5 - x$          $x - y = -1$          $x + y = 7$          $2x + y = 16$          $2x - y = -11$

## 87 Sampling 2

1) The table below shows the number of people belonging to a fitness club.

Age	Gender	Number of Members
< 18	F	45
$18 \leq \text{Age} < 50$	F	233
$\geq 50$	F	115
< 18	M	65
$18 \leq \text{Age} < 50$	M	265
$\geq 50$	M	77

A survey is carried out using a stratified sample of 100 people.

Calculate the number of people selected from each of the six groups.

2) The number of students in the six faculties at a university are shown in the table below.

Faculty	Number of Students	Stratified Sample
Arts	1085	47
Business		16
Education	139	6
Engineering	715	
Law	69	3
Science		27

Some of the figures have been left out.

The administrators decide to ask a sample of the students about the amenities within the college. They ask a stratified sample.

a) Complete the table.

b) How many students are at the university and how many are in the sample?

3) A car-sales company want to ask a sample of their customers about the service the showroom gives. They decide to send letters to 50 of the customers from last year. There were 660 car sales last year. The table below shows the number of cars sold in the different categories.

Type of car	Number sold	Stratified Sample
Mini	226	
Small Family	172	
Large Family	121	
Executive	85	
Estate	34	
4x4	22	

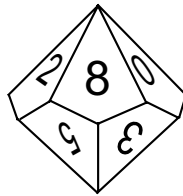
If the sample they use is stratified, calculate the number of people they need to choose from each category of car.

### 88 Sampling 3

Use this table of random 2 digit numbers to answer the questions below.

11	55	59	38	95	33	25	34	14	74
39	63	20	84	96	85	68	98	66	43
71	26	45	15	48	42	35	37	30	07
10	91	23	50	83	21	02	86	76	72
78	62	01	65	57	67	03	51	87	18

- 1) a) Explain how you could produce a table of random numbers, similar to the one above, using 10 sided dice like the one below.



- b) How would you produce a table of 3 figure random numbers?

- 2) The table below shows the number of people belonging to a fitness club.

Age	Gender	Number of Members
< 18	F	45
$18 \leq \text{Age} < 50$	F	233
$\geq 50$	F	115
< 18	M	65
$18 \leq \text{Age} < 50$	M	265
$\geq 50$	M	77

- a) A sample of 10 males aged 50 or more is needed for a survey. Describe how you would use the table of random numbers above to do it.
- b) Which members would you choose?
- c) Explain why you cannot use this table to choose a sample from the 18 to 50 male age group.
- d) Explain what you would need to choose a sample from the 18 to 50 age group.

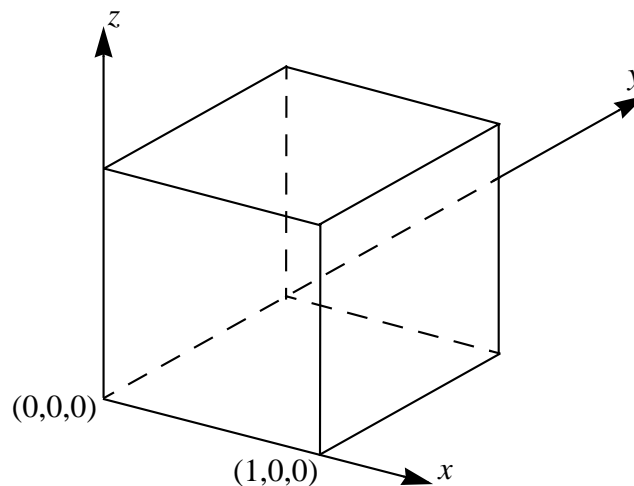
### 89 3 Dimensional Co-ordinates 1

1) Point A has co-ordinates of (7, 9, 11).

Point B has co-ordinates of (1, 3, 5).

What are the co-ordinates of point P, the mid point of line AB?

2) a) The cube shown below has two corners of (0,0,0) and (1,0,0). What are the co-ordinates of the other corners?



b) The cube is moved +2 units in each direction.

(i) What are the new co-ordinates of the corners?

(ii) What are the co-ordinates of the centre of the cube?

3) The co-ordinates of four of the corners of a cube are (0,0,0), (0,0,2), (0,2,2) and (2,2,2).

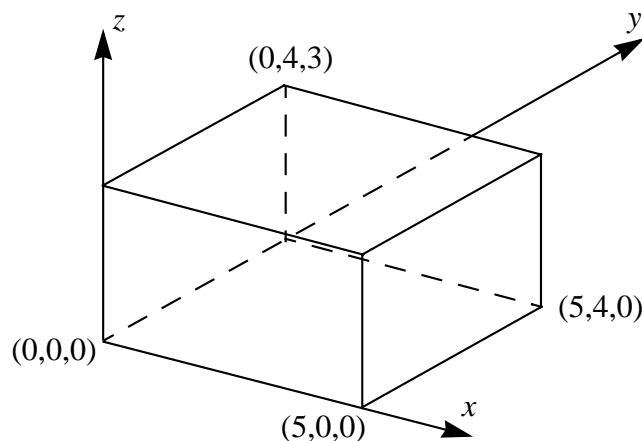
What are the co-ordinates of the other corners?

4) A cube has co-ordinates of (1,1,2), (5,1,2), (5,5,2), (1,5,2), (1,1,6), (5,1,6), (5,5,6) and (1,5,6).

a) Write down the co-ordinates of the centres of each of its six faces.

b) What are the co-ordinates of the centre of the cube?

5) a) The cuboid shown below has four corners of (0,0,0), (5,0,0), (5,4,0), and (0,4,3). What are the co-ordinates of the other corners?



b) What are the co-ordinates of the centres of each of its faces?

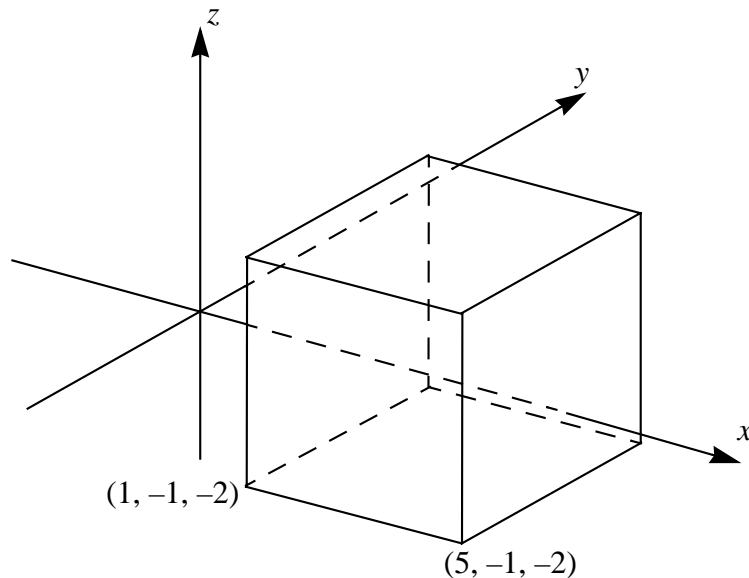
### 90 3 Dimensional Co-ordinates 2

1) Point A has co-ordinates of  $(-5, -3, -1)$ .

Point B has co-ordinates of  $(7, 3, 5)$ .

What are the co-ordinates of point P, the mid point of line AB?

2) a) The cube shown below has two corners of  $(1, -1, -2)$  and  $(5, -1, -2)$ . What are the co-ordinates of the other corners?



b) The cube is moved +2 units in each direction

(i) What are the new co-ordinates of the corners?

(ii) What are the co-ordinates of the centre of the cube?

3) The co-ordinates of four of the corners of a cube are  $(-3, -2, -1)$ ,  $(-3, -2, 1)$ ,  $(-3, 0, 1)$  and  $(-1, 0, 1)$ . What are the co-ordinates of the other corners?

4) A cube has co-ordinates of  $(-2, -3, -2)$ ,  $(-2, -3, 4)$ ,  $(-2, 3, 4)$ ,  $(-2, 3, -2)$ ,  $(4, -3, -2)$ ,  $(4, 3, -2)$ ,  $(4, 3, 4)$  and  $(4, -3, 4)$ .

a) What is the length of one side of the cube?

b) Write down the co-ordinates of the centres of each of its six faces.

c) What are the co-ordinates of the centre of the cube?

d) If the cube is repositioned so that corner  $(-2, -3, -2)$  is moved to  $(0, 0, 0)$ , what is the new position of corner  $(4, 3, -2)$

5) A cuboid has four corners of  $(-7, -6, -5)$ ,  $(-7, -6, -3)$ ,  $(-7, -3, -5)$ , and  $(-3, -3, -3)$ .

a) What are the co-ordinates of the other corners?

b) What are the co-ordinates of the centres of each of its faces?

c) What are the co-ordinates of the centre of the cuboid?

d) What are the dimensions of the cuboid?

e) If corner  $(-7, -6, -5)$  is moved to position  $(0, 0, 0)$  to what position will corner  $(-3, -3, -3)$  move?



## 91 Moving Averages

The table below shows the value of the shares of a company over a period of 24 weeks. The value of each share is taken at the end of each week. A shareholder needs to know the trend in the share value of the company and does this by calculating an 8 week moving average.

Day	Value of a share in pence	8 week moving average
1	251	
2	263	
3	294	
4	330	
5	350	
6	345	
7	342	
8	340	
9	336	
10	331	
11	330	
12	321	
13	315	
14	302	
15	321	324.5
16	317	
17	320	
18	318	
19	307	
20	301	
21	331	
22	287	
23	305	
24	306	

- b) Explain why there can be no moving average for week 5.
- c) When can the first moving average be calculated?
- d) Complete the table of moving averages.
- e) By looking at the moving averages, what do you think the trend in his share values is?

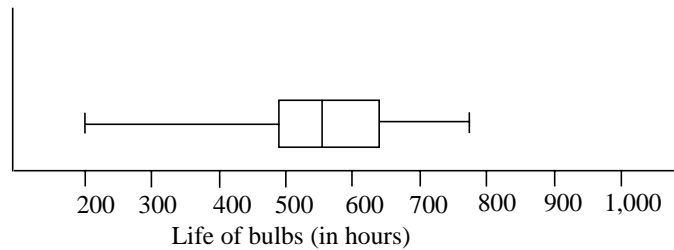
### 92 Box Plots

1) There are 150 apple trees in the cider farmer’s orchard. The following figures relate to their heights.

Tallest tree 450cm	Smallest tree 250cm
Lower quartile 310cm	Upper quartile 360cm
Median 334cm	

Represent this information on a box plot.

2) The box plot below represents the life of 30 Regular electric light bulbs tested by the Shiny Electric Light Company.

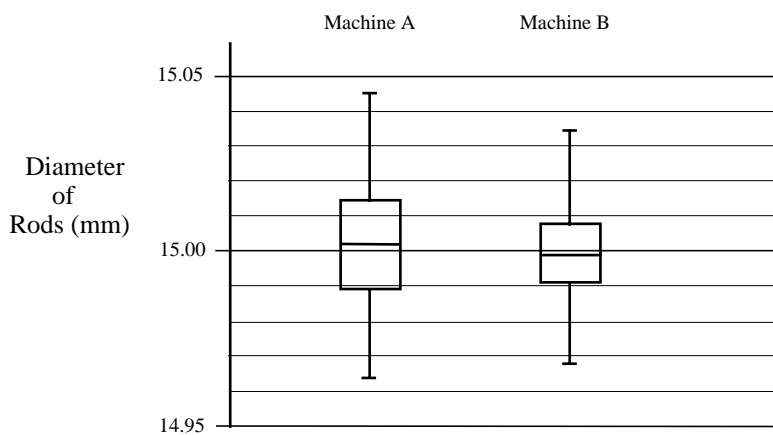


When they test the Super bulbs they get the following results.

Shortest life 350 hours	Longest life 950 hours
Lower quartile 650 hours	Upper quartile 770 hours
Median life 720 hours.	

Copy the diagram and on it show the box plot for the Super bulbs.  
Compare the two types of bulbs and make comments.

3) The diagram shows two box plots representing the diameters of rods made on two machines. Two samples of 100 rods were taken from each of the machines.



The rods are produced to a diameter of 15.00 mm and any rod which is more than 0.01mm above or below that value is a) scrapped if it is too small or b) re-machined if it is too large. Any value between these allowances is acceptable.

Use the diagrams to comment on the accuracy of the machines.

**Answers****1. Estimations and Calculation**

- 1) a) 1.857      b)  $\frac{2 \times 5}{5}$       c) 2  
 2) a) 20.12      b)  $\frac{10 \times 4}{2}$       c) 20  
 3) a) 18.02      b)  $\frac{9 \times 9}{4}$       c) 20  
 4) a) 1.002      b)  $\frac{0.5 \times 7}{3}$       c) 1  
 5) a) 0.1074      b)  $\frac{3 \times 0.4}{9}$       c) 0.1  
 6) a) 0.3274      b)  $\frac{8 \times 0.3}{8}$       c) 0.3  
 7) a) 0.1099      b)  $\frac{0.7 \times 0.8}{5}$       c) 0.1  
 8) a) 0.01814      b)  $\frac{0.2 \times 0.6}{6}$       c) 0.02  
 9) a) 0.06096      b)  $\frac{0.6 \times 0.8}{8}$       c) 0.06  
 10) a) 1.035      b)  $\frac{2 \times 0.2}{0.4}$       c) 1  
 11) a) 12.22      b)  $\frac{8 \times 0.3}{0.2}$       c) 12  
 12) a) 4.332      b)  $\frac{20 \times 0.2}{0.8}$       c) 5  
 13) a) 616.3      b)  $\frac{10 \times 30}{0.6}$       c) 500  
 14) a) 6500      b)  $\frac{30 \times 100}{0.4}$       c) 7500  
 15) a) 21080      b)  $\frac{200 \times 30}{0.3}$       c) 20000  
 16) a) 0.9753      b)  $\frac{20 \times 0.02}{0.3}$       c) 1  
 17) a) 1445      b)  $\frac{30 \times 0.8}{0.02}$       c) 1200  
 18) a) 1,411,000      b)  $\frac{70 \times 30}{0.001}$       c) 2,100,000  
 19) a) 395.9      b)  $\frac{(30)^2 \times 10}{30}$       c) 300  
 20) a) 5.303      b)  $\frac{5 \times (0.8)^2}{0.6}$       c) 5  
 21) a) 58.88      b)  $\frac{20 \times (20)^2}{100}$       c) 80  
 22) a) 246.9      b)  $\frac{(20)^2 \times 2}{0.6 \times 3}$       c) 400  
 23) a) 0.5510      b)  $\frac{100 \times (0.3)^2}{0.8 \times 30}$       c) 0.4  
 24) a) 0.7783      b)  $\frac{(0.3)^2 \times 30}{0.5 \times 7}$       c) 0.8  
 25) a) 13.36      b)  $\frac{40 \times 7}{20}$       c) 14  
 26) a) 6.883      b)  $\frac{10 \times 20}{50}$       c) 4

- 27) a) 2.099      b)  $\frac{30 \times 0.9}{10}$       c) 2.7  
 28) a) 16.89      b)  $\frac{30^2 \times 0.4}{20}$       c) 18  
 29) a) 1231      b)  $\frac{40 \times 10}{0.4}$       c) 1000  
 30) a) 639.9      b)  $\frac{30^2 \times 20}{3 \times 10}$       c) 600  
 31) a) 7009      b)  $\frac{30 \times 20^2}{0.3 \times 4}$       c) 10,000  
 32) a) 1372      b)  $\frac{30^2 \times 0.8^2}{0.4}$       c) 1440  
 33) a) 213.1      b)  $\frac{40 \times 4^2}{0.3 \times 10}$       c) 160  
 34) a) 8.471      b)  $\frac{10 \times 30}{40}$       c) 7.5  
 35) a) 2455      b)  $\frac{40^2 \times 0.02}{0.8 \times 0.01}$       c) 4000  
 36) a) 0.1352      b)  $\frac{0.5^2 \times 0.006}{0.8 \times 0.02}$       c) 0.09  
 37) a) 6784      b)  $\frac{100}{0.02}$       c) 5000  
 38) a) 5.866      b)  $\frac{27000 \times 0.008}{32}$       c) 7

**2. Standard Form****Exercise 1**

- 1)  $4.57 \times 10^2$       2)  $1.427 \times 10^3$       3)  $9.431 \times 10^3$   
 4)  $1.56321 \times 10^5$       5)  $1.7 \times 10^7$       6)  $2.813 \times 10^{-1}$   
 7)  $8.142 \times 10^{-2}$       8)  $4.86 \times 10^{-4}$       9)  $9.7 \times 10^{-6}$

**Exercise 2**

- 1) 280,000      2) 64,000,000      3) 93,000  
 4) 4,315,000      5) 8,614,000,000      6) 0.0431  
 7) 0.0000032      8) 0.000000684      9) 0.00000000438

**Exercise 3**

- 1)  $2.432 \times 10^7$       2)  $4.482 \times 10^9$       3)  $1.564 \times 10^{-4}$   
 4)  $3.784 \times 10^{-1}$       5)  $5.312 \times 10^7$       6)  $2.438 \times 10^{-3}$   
 7)  $1.676 \times 10^{-4}$       8)  $1.537 \times 10^{-9}$       9)  $3.5 \times 10^8$   
 10)  $1.25 \times 10^{-5}$       11)  $1.37 \times 10^{10}$       12)  $8.123 \times 10^4$   
 13)  $1.316 \times 10^8$       14)  $2.400 \times 10^{-4}$       15)  $1.207 \times 10^4$   
 16)  $2.093 \times 10^2$       17)  $9.928 \times 10^5$       18)  $1.963 \times 10^{-14}$   
 19)  $2 \times 10^2$       20)  $3 \times 10^{-3}$

**Exercise 4**

- 1) a)  $9.4608 \times 10^{12}$  km      b) 8.456 years  
 2) a)  $5.976 \times 10^{21}$  and  $7.35 \times 10^{19}$       b) 81 times  
 3) a) 1,839      b)  $1.098 \times 10^{30}$  electrons

**3. Rational & Irrational Numbers****Exercise 1**

- 1)  $\frac{23}{90}$       2)  $\frac{14}{45}$       3)  $\frac{49}{90}$   
 4)  $\frac{28}{45}$       5)  $\frac{2}{45}$       6)  $\frac{11}{15}$   
 7)  $\frac{7}{900}$       8)  $\frac{4}{225}$       9)  $\frac{5}{33}$   
 10)  $\frac{32}{99}$       11)  $\frac{53}{99}$       12)  $\frac{4}{55}$

**Exercise 2**

1, 2, 7, 9, 10, 12, 13, 15

**Exercise 3**

2, 3, 4, 5, 6, 8, 9, 14, 15, 16

**Exercise 4**

1, 3, 4, 5, 6, 8, 11

**Exercise 5**

c, e

**4. Surds****Exercise 1**

- 1)  $2\sqrt{2}$       2)  $2\sqrt{3}$       3)  $2\sqrt{6}$       4)  $2\sqrt{7}$   
 5)  $6\sqrt{3}$       6)  $2\sqrt{10}$       7)  $5\sqrt{2}$       8)  $3\sqrt{2}$   
 9)  $4\sqrt{3}$       10)  $4\sqrt{2}$       11)  $2\sqrt{5}$       12)  $5\sqrt{5}$   
 13)  $10\sqrt{2}$       14)  $6\sqrt{6}$       15)  $8\sqrt{3}$       16)  $8\sqrt{5}$

**Exercise 2**

- 1)  $\sqrt{2}$       2)  $\sqrt{3}$       3) 2      4)  $3\sqrt{2}$   
 5)  $2\sqrt{7}$       6)  $4\sqrt{2}$       7)  $3\sqrt{3}$       8)  $4\sqrt{3}$   
 9)  $7\sqrt{2}$       10)  $10\sqrt{2}$       11)  $10\sqrt{3}$       12)  $10\sqrt{5}$   
 13)  $14\sqrt{5}$       14)  $13\sqrt{3}$       15)  $7\sqrt{7}$       16)  $3\sqrt{21}$

**Exercise 3**

- 1)  $3\sqrt{2}$       2)  $4\sqrt{3}$       3)  $5\sqrt{2}$       4)  $3\sqrt{2}$   
 5)  $\sqrt{2}$       6)  $\sqrt{3}$       7)  $\sqrt{5}$       8)  $2\sqrt{2}$   
 9)  $\sqrt{5}$       10)  $\sqrt{5}$       11)  $2\sqrt{7}$       12)  $7\sqrt{5}$

**Exercise 4**

- 1)  $\sqrt{2}$       2)  $\frac{\sqrt{2}}{2}$       3)  $3\sqrt{3}$       4)  $8\sqrt{2}$   
 5)  $8\sqrt{3}$       6)  $\sqrt{7}$       7)  $7\sqrt{5}$       8)  $\sqrt{3}$   
 9)  $4\sqrt{7}$       10)  $2\sqrt{5}$       11)  $9\sqrt{5}$       12)  $18\sqrt{2}$

**Exercise 5**

- 1)  $3\sqrt{2}$       2)  $5\sqrt{2}$       3) 6      4)  $6\sqrt{2}$   
 5) 14      6) 24      7)  $16\sqrt{6}$       8)  $60\sqrt{2}$   
 9) 48      10)  $20\sqrt{5}$       11)  $42\sqrt{6}$       12)  $112\sqrt{6}$

**5. Prime Factors****Exercise 1**

- 1)  $2^2 \times 3 \times 5^2$       2)  $2^2 \times 3^2 \times 5^2$       3)  $2 \times 3^2 \times 5 \times 7$   
 4)  $2^2 \times 5^2 \times 7$       5)  $2^3 \times 3^2 \times 11$       6)  $3^3 \times 5 \times 7$   
 7)  $2^3 \times 5 \times 7^2$       8)  $3 \times 5 \times 11^2$       9)  $2^3 \times 3^3 \times 7$   
 10)  $2^2 \times 3 \times 5 \times 11 \times 13$       11)  $2^4 \times 3 \times 5 \times 11$   
 12)  $2^2 \times 3 \times 5 \times 7 \times 13$       13)  $2^5 \times 3^2 \times 13$

14)  $2^6 \times 3^2 \times 11$

15)  $2^3 \times 3 \times 5 \times 7 \times 11$

**Exercise 2**

- 1)  $2^2 \times 3 \times 5 \times 11$       165      2)  $2^2 \times 3 \times 5^2$       3  
 3)  $2 \times 3^2 \times 5^2$       2      4)  $2^2 \times 5^2 \times 7$       7  
 5)  $3^2 \times 5^2 \times 7$       7      6)  $3^2 \times 5 \times 7^2$       5  
 7)  $2^3 \times 3 \times 5^2$       6      8)  $2^2 \times 3^2 \times 11$       11  
 9)  $2 \times 3^3 \times 5^2$       6      10)  $2^4 \times 3^2 \times 13$       13  
 11)  $2 \times 3^2 \times 5^2 \times 11$       22      12)  $2^8 \times 3 \times 5$       15  
 13)  $2^2 \times 3^2 \times 5 \times 7^2$       5      14)  $2^4 \times 3 \times 5 \times 7^2$       15  
 15)  $2^2 \times 3^4 \times 5 \times 7$       35

**Exercise 3**

- 1) 15      2) 105      3) 27  
 4) 13      5) 55      6) 11  
 7) 143      8) 75      9) 115  
 10) 231      11) 77      12) 33  
 13) 735      14) 4725      15) 1155

**Exercise 4**

- 1) a) 21      b)  $2.1\text{cm} \times 2.1\text{cm}$       c) 1260  
 2) a) 15      b) 15  
 3) a) 18      b) 180 boxes      c) 55 and 42

**6. Percentages**

- 1) £2000  
 2) 15%  
 3) a) £282      b) £24      c) £850  
 4) a) £97.50      b) £21.45  
 5) a) £24,725      b) 10.4%  
 6) 61,252  
 7) 20.5%  
 8) £5222.25  
 9) a) 800ml      b) Yes - 20% increase in cola but a 33% increase in price.  
 10) a) 39%      b) 4096  
 11) 57%  
 12) 6 years  
 13) 546,250  
 14) 6 years  
 15) A by £315  
 16) £1116.25

**7. Conversion Graphs**

- 1) a) 2,880      b) 62,000      c) £49  
 2) a) £17.20      b) 1m 60cm  
 3) b) 42cm      c) £27

**8. Fractions****Exercise 1**

- 1)  $\frac{3}{5}$       2) 1      3)  $1\frac{2}{5}$   
 4)  $\frac{8}{15}$       5)  $\frac{15}{56}$       6)  $\frac{1}{12}$   
 7)  $1\frac{4}{15}$       8)  $\frac{18}{35}$       9)  $1\frac{13}{36}$

- 10)  $1\frac{3}{20}$       11)  $1\frac{23}{24}$       12)  $3\frac{22}{35}$   
 13)  $\frac{3a}{7}$       14)  $\frac{6a}{5}$       15)  $a$   
 16)  $\frac{8x}{15}$       17)  $\frac{12x}{35}$       18)  $\frac{3b}{10}$   
 19)  $\frac{x}{8}$       20)  $\frac{29x}{70}$       21)  $\frac{67x}{90}$   
 22)  $\frac{9a}{4}$       23)  $\frac{37x}{14}$       24)  $\frac{117x}{44}$

**Exercise 2**

- 1)  $\frac{5}{x}$       2)  $\frac{1}{a}$       3)  $\frac{15}{x}$   
 4)  $\frac{5}{2x}$       5)  $\frac{4}{x}$       6)  $\frac{11}{20x}$   
 7)  $\frac{13}{6x}$       8)  $\frac{9}{20a}$       9)  $\frac{a+b}{ab}$   
 10)  $\frac{2y-x}{xy}$       11)  $\frac{3y+5x}{xy}$       12)  $\frac{b+5}{5b}$   
 13)  $\frac{2b+15}{5b}$       14)  $\frac{4+x}{5}$       15)  $\frac{16-x^2}{4x}$   
 16)  $\frac{x^2-12}{4x}$

**Exercise 3**

- 1)  $\frac{3a+11}{4(a+1)}$       2)  $\frac{5x-3}{8(x+1)}$   
 3)  $\frac{3x-23}{10(x-1)}$       4)  $\frac{7x-3}{12}$   
 5)  $\frac{3a-4(x+1)}{12}$       6)  $\frac{8b+15a}{20}$   
 7)  $\frac{53x+6}{18}$       8)  $\frac{33a+12}{20}$   
 9)  $\frac{41x+32}{24}$       10)  $\frac{2-x}{6}$   
 11)  $\frac{14y+15}{30}$       12)  $\frac{61a+20}{28}$   
 13)  $\frac{9(x-3)}{(x-1)(x-4)}$       14)  $\frac{18x+1}{(x+2)(2x-1)}$   
 15)  $\frac{14x+1}{(3x+2)(4x+1)}$       16)  $\frac{15+2xy}{6x}$   
 17)  $\frac{19x}{5a}$       18)  $\frac{7x}{3a}$   
 19)  $\frac{1}{30}(x-9)$       20)  $\frac{1}{40}(9x+73)$   
 21)  $\frac{1}{35}(36a-57)$       22)  $\frac{7x+9}{4}$   
 23)  $\frac{a+30}{4}$       24)  $\frac{3(x+11)}{4}$   
 25)  $\frac{x^2+2}{x}$       26)  $\frac{13a+1}{3}$       27)  $\frac{17x-3}{4}$

**9. Indices****Exercise 1**

- 1)  $x^7$       2)  $y^{11}$       3)  $a^7$       4)  $b^{12}$   
 5)  $12x^2$       6)  $5a^7$       7)  $24y^5$       8)  $35x^{11}$

- 9)  $a^3$       10)  $b^4$       11)  $y^2$       12)  $y^5$   
 13)  $a^6$       14)  $18$       15)  $2^8$       16)  $12$   
 17)  $x^3y^3$       18)  $a^7b^6$       19)  $x^4y^8$       20)  $a^7b^7$   
 21)  $16x^2$       22)  $9x^4$       23)  $16y^6$       24)  $16a^8$   
 25)  $3a^2$       26)  $18x^3$       27)  $4y^3$       28)  $4a^3$   
 29)  $4a^2$       30)  $6b^6$       31)  $4a^2b$       32)  $3xy$   
 33)  $x^4$       34)  $6a^8$       35)  $14y^4$       36)  $3x^8$

**Exercise 2**

- 1)  $x^0$  or 1      2)  $a^{-2}$       3)  $2y^{-1}$       4)  $\frac{1}{2b^2}$   
 5)  $\frac{1}{y}$  or  $y^{-1}$       6)  $\frac{6}{x^3}$       7)  $\frac{12}{a^3}$       8)  $\frac{6a^3}{b}$   
 9)  $x^3$       10)  $\frac{1}{y^3}$       11)  $6x^2$       12) 24  
 13)  $a$       14) 1      15)  $b^{\frac{1}{2}}$       16)  $y^{-1}$   
 17) 1      18)  $y^{\frac{1}{4}}$       19)  $a^{\frac{3}{4}}$       20)  $b^{\frac{1}{2}}$   
 21)  $x^{\frac{3}{2}}$       22)  $a^{\frac{1}{2}}$       23)  $8b$   
 24)  $\sqrt{5y^{\frac{3}{2}}}$  or  $\sqrt{5y^3}$

**Exercise 3**

- 1) 5      2)  $\frac{1}{5}$       3) 2      4)  $\frac{1}{3}$   
 5) 4      6) 8      7) 8      8) 27  
 9) 27      10) 16      11) 8      12) 125  
 13)  $\frac{1}{125}$       14)  $\frac{1}{32}$       15)  $\frac{5}{2}$       16)  $\frac{25}{4}$

**Exercise 4**

- 1)  $x = 2$       2)  $x = 2$       3)  $x = 4$       4)  $x = 3$   
 5)  $x = \frac{1}{5}$       6)  $x = -\frac{1}{2}$       7)  $x = 2$       8)  $x = 3$   
 9)  $x = 5$       10)  $x = 81$       11) 16      12)  $x = -\frac{1}{2}$

**10. Simplifying****Exercise 1**

- 1)  $4x$       2)  $-2y$       3)  $-5y$   
 4)  $-13x$       5)  $9x+9y$       6)  $3x-2y$   
 7)  $5y-10x$       8)  $-4x-10y$       9)  $3b-4a-4ab$   
 10)  $12b-11a+3ab$       11)  $2x^2+3x$       12)  $y^2-4y$   
 13)  $4y^2$       14)  $3y^2-10x^2$       15)  $3x+4y$   
 16)  $10x-3y$       17)  $7x+4y$       18)  $4x-6y$   
 19)  $6y-2x$       20)  $8y-2x$       21)  $5x+y$   
 22)  $7x+10y$       23)  $8x-14y$       24)  $-y$   
 25)  $3x+3$       26)  $-5x-9y$       27)  $2x^2-24x$   
 28)  $18y-14x-5xy$       29)  $8y^2-6xy-2x^2$   
 30)  $4xy+12x^2+9y^2$       31)  $\frac{7x}{12}$   
 32)  $\frac{x}{6}$       33)  $\frac{y}{6}$       34)  $\frac{y^2}{6}$

**Exercise 2**

- 1)  $x^2-2x-3$       2)  $x^2-2x-8$

- 3)  $2x^2 - 11x - 21$       4)  $6x^2 + 11x - 10$   
 5)  $6x^2 - 17x - 14$       6)  $6x^2 - 7x - 20$   
 7)  $35x^2 - x - 6$       8)  $12x^2 + 16x - 16$   
 9)  $8x^2 - 4x - 4$       10)  $6x^2 + 13xy + 6y^2$   
 11)  $4x^2 + 13xy + 3y^2$       12)  $2x^2 - 14x + 24$   
 13)  $x^2 + 2x + 1$       14)  $9x^2 + 12x + 4$   
 15)  $25x^2 - 20x + 4$       16)  $9x^2 + 24x + 16$   
 17)  $25x^2 - 60x + 36$       18)  $49x^2 + 28x + 4$   
 19)  $9x^2 - 12x + 4$       20)  $25x^2 + 70x + 49$   
 21)  $16x^2 + 48x + 36$       22)  $x^2 + 2xy + y^2$   
 23)  $4x^2 + 12xy + 9y^2$       24)  $16x^2 - 16xy + 4y^2$

**11. Re-arranging Formulae****Exercise 1**

- 1)  $v - at$       2)  $\frac{v-u}{t}$       3)  $\frac{d+c}{3}$   
 4)  $c - pd$       5)  $7y - x$       6)  $\frac{a-c}{3}$   
 7)  $\frac{3w-u}{4}$       8)  $4x - 5y$       9)  $x$   
 10)  $\frac{8y}{3}$       11)  $2(p - 3b)$       12)  $\frac{w}{2} - \frac{u}{8}$  or  $\frac{1}{2}(w - \frac{1}{4}u)$   
 13)  $\frac{a+b}{c}$       14)  $\frac{2r-q}{3p}$       15)  $\frac{z}{5} - y$   
 16)  $\frac{1}{4}(\frac{a}{3} - 3b)$       17)  $2x - z$       18)  $9a - 2b$   
 19)  $y - 12x$       20)  $\frac{3v-15w}{2}$       21)  $\frac{14}{11}y$   
 22)  $-\frac{3}{19}b$

**Exercise 2**

- 1)  $\sqrt{ac}$       2)  $\sqrt{\frac{y}{x}}$   
 3)  $\frac{\sqrt{bc}}{2}$       4)  $\sqrt{\frac{3}{v}}$   
 5)  $y = -\frac{x}{6}$       6)  $y = -8x$   
 7)  $x = \frac{y}{6}$       8)  $b = -\frac{27}{7}c$   
 9)  $x = \frac{ab}{b+a}$       10)  $x = \frac{2yz}{3(2z+3y)}$   
 11)  $x = \left(\frac{6ab}{3b+2a}\right)^2$       12)  $x = \left(\frac{4y}{3+by}\right)^2$   
 13)  $z = \frac{6x}{3xy-4}$       14)  $z = \frac{xy}{3(2x-y)}$   
 15)  $a = \sqrt{\frac{b}{xb-1}}$       16)  $a = \sqrt{\frac{2}{3(4y-3b)}}$   
 17)  $b = 2c$       18)  $x = \frac{y}{3}$   
 19)  $y = \frac{z}{3x-2}$       20)  $c = \frac{b}{4a+3}$   
 21)  $x = 1 - 3y$       22)  $y = \frac{x-4}{3}$

**12. Number Sequences****Exercise 1**

- 1) 20, 23      2) 25, 29      3) 25, 32  
 4) 17, 23      5) 44, 52      6) -1, -8  
 7) -2, -4      8) -21, -34      9) -3, -8  
 10) 128, 256      11) -22, -29      12) -21, -27  
 13) 29, 35      14) -6, -10      15) 127, 255  
 16) 49, 64      17) -5, -7      18) 343, 512  
 19) 335, 504      20) 46, 61      21) 64, 128  
 22) 146, 191      23) 8, 13      24) 86, 139  
 25) 28, 36      26) 49, 60

**Exercise 2**

- 1) 22, 25       $3n + 1$       2) 27, 32       $5n - 8$   
 3) -26, -30       $2 - 4n$       4) -22, -27       $13 - 5n$   
 5) -9, -17       $47 - 8n$       6) 36, 41       $5n + 1$   
 7) 23, 30       $7n - 26$       8) 49, 64       $n^2$   
 9) 51, 66       $n^2 + 2$       10) 42, 57       $n^2 - 7$   
 11) 36, 49       $(n-1)^2$       12) 42, 56       $n^2 - n$   
 13)  $\frac{14}{15}, \frac{16}{17}$        $\frac{2n}{2n+1}$       14)  $\frac{7}{8}, \frac{8}{9}$        $\frac{n}{n+1}$   
 15)  $\frac{49}{10}, \frac{64}{11}$        $\frac{n^2}{n+3}$       16)  $\frac{21}{97}, \frac{24}{127}$        $\frac{3n}{2n^2-1}$

**Exercise 3**

- 1) a)  $n + 1$   
 b)  $(n+1)(y+1) = ny + n + y + 1 = (ny + n + y) + 1$   
 which is a term in the sequence  
 2) a)  $(2n+1)$   
 b)  $(2n+1)(2y+1) = 4ny + 2n + 2y + 1$   
 $= 2(2ny + n + y) + 1$  which is a term.  
 3) a)  $n^2$       b)  $n^2 y^2 = (ny)^2$  which is a term.  
 4) a)  $4n + 1$       b)  $4(4ny + n + y) + 1$  is a term.

**13. Substitution****Exercise 1**

- 1) 38      2) 0      3) -16      4) -8  
 5) 50      6) 116      7) 52      8) 105  
 9) 2116      10) 156      11) -100      12) -589

**Exercise 2**

- 1) a) 22      b)  $x = 5$   
 2) a) -27      b)  $b = 20$   
 3) a) -39      b)  $x = -12$   
 4) a) 700 cm      b)  $R = 26$   
 5) a)  $23.565 \text{ cm}^3$       b)  $7 \text{ cm}$   
 6) a) £71.25      b) £476.19  
 7) a)  $-29^\circ$       b)  $F = 14^\circ$   
 8) a) 55 minutes      b) 1 hour 19 minutes  
 c) Decrease the numbers that D and S are divided by.

**14. Factorising****Exercise 1**

- 1)  $4(x+2)$       2)  $3(2y-3)$       3)  $7(b-2a)$   
 4)  $x(y-1)$       5)  $x(3+y)$       6)  $2y(2+5x)$   
 7)  $2(3x^2+1)$       8)  $x(5x-1)$       9)  $3x(3x-1)$   
 10)  $ab(a+b)$       11)  $ab(4-a)$       12)  $2ab(4+3b)$

- 13)  $a(1+b-a)$  14)  $a(3b-c+a)$  15)  $y(5x^2-4y-3x)$   
 16)  $\frac{x^2}{4}(2-x)$  17)  $\frac{y}{6}(2y-x)$  18)  $\frac{x}{6}(5x-4)$

**Exercise 2**

- 1)  $(m-n)(m+n)$  2)  $(a-2)(a+2)$   
 3)  $(xy-z)(xy+z)$  4)  $(ab-3)(ab+3)$   
 5)  $(xy-2)(xy+2)$  6)  $(vw-5)(vw+5)$   
 7)  $(ab-3c)(ab+3c)$  8)  $(5a-3b)(5a+3b)$   
 9)  $(b-1)(b+1)$  10)  $2(a-5)(a+5)$   
 11)  $2(2a-5)(2a+5)$  12)  $3(2x-3y)(2x+3y)$   
 13)  $x(y-2x)(y+2x)$  14)  $2x(y-2x)(y+2x)$   
 15)  $x^2(2y-3)(2y+3)$  16)  $(x-y)(x+y)(x^2+y^2)$   
 17)  $(2x-3y)(2x+3y)(4x^2+9y^2)$   
 18)  $3(a^2-2b)(a^2+2b)$

**Exercise 3**

- 1)  $(x+3)(x+1)$  2)  $(x+2)(x+2)$   
 3)  $(x+7)(x+1)$  4)  $(x+5)(x+2)$   
 5)  $(x+4)(x+3)$  6)  $(x+6)(x+5)$   
 7)  $(x-1)(x+3)$  8)  $(x-3)(x+1)$   
 9)  $(x+5)(x-1)$  10)  $(x+2)(x-4)$   
 11)  $(x-5)(x+3)$  12)  $(x+3)(x-4)$   
 13)  $(x-6)(x-4)$  14)  $(x-3)(x-5)$   
 15)  $(x-4)(x-7)$

**Exercise 4**

- 1)  $(2x+1)(x+1)$  2)  $(2x+1)(x+4)$   
 3)  $(2x+1)(x+3)$  4)  $(2x+2)(x+3)$   
 5)  $(2x-3)(x+2)$  6)  $(3x+2)(x-3)$   
 7)  $(2x-1)(x-4)$  8)  $(3x-1)(x-3)$   
 9)  $(3x-2)(x-4)$  10)  $(3x+7)(x-2)$   
 11)  $(3x+4)(x+5)$  12)  $(3x-6)(x-2)$   
 13)  $(2x+2)(2x+3)$  14)  $2(x-1)(2x-3)$   
 15)  $(4x+1)(x+3)$  16)  $(x+5)(4x+1)$   
 17)  $(5x-2)(x+3)$  18)  $(3x+2)(2x-3)$   
 19)  $(3x+1)(2x+1)$  20)  $(3x+2)(3x+2)$   
 21)  $(x+1)(8x+3)$  22)  $(x-5)(4x-3)$   
 23)  $(5x+2)(x-3)$  24)  $(4x-3)(3x-1)$

**15. Trial and Improvement****Exercise 1**

- 1) 3.8 2) 4.3 3) 4.7  
 4) 5.5 5) 4.3 6) 6.5  
 7) 5.2 8) 3.8 9) 2.4  
 10) 2.1 11) 4.8 12) 2.9  
 13) 3.5 14) 1.9 15) 1.9

**Exercise 2**

- 1) 6.7 2) 8.2 3) 9.4  
 4) 3.3 5) 3.5 6) 1.5  
 7) 2.4 8) 3.3

**16. Iteration**

- 1) 3.73 2) 5.30 3) 3.79  
 4) 8.77 5) 0.62 6) -6.6  
 7) a) (i) -3.50, -2.80, -3.09, -2.96 (ii) -3  
 (iii)  $9+12-21=0$   
 8) a) (i) 7.4, 6.946, 7.008 (ii) 7.0 b)  $49-42-7=0$

**17. Direct and Inverse Proportion**

- 1) a)  $y = \frac{2}{3}x$  b)  $y = 5\frac{1}{3}$  c)  $x = 22\frac{1}{2}$   
 2)  $v = 81$   
 3) a)  $y = \frac{2}{3}\sqrt{x}$  b)  $y = \frac{4}{3}$  c)  $x = 81$   
 4)  $a = 256$   
 5) a)  $y = \frac{3}{x^2}$  b)  $y = \frac{1}{3}$  c)  $x = 6$   
 6)  $y = \frac{3}{4}$   
 7) a)  $y = 5\sqrt[3]{x}$  b) i)  $y = 20$  ii)  $x = 125$   
 8)

$x$	1	2	3	4
$y$	3	0.75	0.3	0.1875

- 9)  $a = \frac{2}{3}$   
 10)

$x$	0.125	1	8	64
$y$	0.25	0.5	1	2

- 11) a)  $n = 3$  b)  $y = 0.125$   
 12)

$x$	0.5	0.75	1	1.5
$y$	6	2.6	1.5	0.6

**18. Solving Equations 1****Exercise 1**

- 1)  $x = 9$  2)  $x = 12$  3)  $x = 6$   
 4)  $x = 4$  5)  $x = 120$  6)  $x = 150$   
 7)  $x = 12$  8)  $x = 8$  9)  $x = 1$   
 10)  $x = \frac{7}{5}$  11)  $x = 1$  12)  $x = 2$   
 13)  $x = -2$  14)  $x = 3$  15)  $x = 5$   
 16)  $x = 8$  17)  $x = 2$  18)  $x = 7$   
 19)  $x = 24$  20)  $x = 32$  21)  $x = 11$   
 22)  $x = 6$  23)  $x = -11$  24)  $x = \frac{4}{5}$   
 25)  $x = -\frac{1}{3}$  26)  $x = \frac{1}{2}$

**Exercise 2**

- 1)  $x = 5$  or  $-5$  2)  $x = 9$  or  $-9$   
 3) 6 or  $-6$  4) 3 or  $-3$   
 5)  $-2$  or 3 6) 6 or  $-5$   
 7)  $-\frac{4}{3}$  or  $\frac{1}{2}$  8) 0 or  $\frac{2}{3}$   
 9) 0 or  $-\frac{3}{4}$  10) 0 or 4  
 11) 0 or  $-\frac{3}{2}$  12) 0 or  $\frac{2}{3}$   
 13) 0 or  $\frac{1}{4}$  14)  $-2$  or 3  
 15)  $\frac{3}{2}$  or  $-1$  16)  $-1$  or  $\frac{1}{3}$   
 17) 5 or  $-2$  18) 2 or  $-4$   
 19)  $-\frac{5}{4}$  or  $\frac{3}{2}$  20) 2.5 or  $-2.5$   
 21)  $-2$  22)  $-\frac{1}{3}$

**19. Solving Equations 2****Exercise 2**

- 1) 1.16 or  $-5.16$  2) 2.62 or 0.38  
 3) 4.65 or  $-0.65$  4) 0.61 or  $-6.61$   
 5) 0.62 or  $-1.62$  6) 2.20 or  $-0.45$   
 7)  $-0.73$  or 0.23 8) 0.74 or  $-0.94$   
 9) 1.08 or  $-2.33$  10) 1.72 or  $-0.39$

- 11) 0.79 or -2.12      12) 0.19 or -2.69  
 13) 1 or 0.75      14) 0.23 or -0.73  
 15) 0.18 or -1.61      16) 3.24 or 0.12  
 17) 0.78 or -0.92      18) 1.92 or 0.46  
 19) 0.37 or -5.37      20) 0.52 or -9.52  
 21) 1.44 or -0.69      22) 2.77 or -1.27  
 23) 2.30 or -1.30      24) 1.81 or -1.47  
 25) 2.78 or 0.72      26) 10.18 or -1.18

**Exercise 2**

- 1)  $a = 5$     $k = 25$       2)  $a = 2$     $k = 4$   
 3)  $a = 3$     $k = 9$       4)  $a = 7$     $k = 49$   
 5)  $a = 1.5$     $k = 2.25$       6)  $a = 2$     $k = 4$   
 7)  $a = 5$     $k = 25$       8)  $a = 8$     $k = 64$   
 9)  $a = 6$     $k = 36$       10)  $a = 4$     $k = 16$

**20. Problems Involving Equations**

- 1) a)  $x = 34^\circ$     $34^\circ, 68^\circ, 78^\circ$   
 b)  $x = 30^\circ$     $90^\circ, 40^\circ, 50^\circ$   
 2) a) 538.46 mph  
 b)  $\frac{x}{538.46}$  hours      c)  $\frac{538.46y}{60}$  miles  
 3) a)  $x = 4.5$  cm      b)  $30.25 \text{ cm}^2$   
 4) a)  $x - 1$  and  $x + 1$       b)  $x = 52$   
 5) a) i)  $x + \frac{y}{4}$       ii)  $\frac{3y}{4}$   
 b)  $y = 8x$       c) 18,000 bottles  
 6) a)  $250 - x$       b)  $x = 110$       c) 140  
 7) a)  $x - 6$  and  $x - 21$       b)  $3x - 27$       c)  $x = 40$   
 34, 40 and 19

**21. Simultaneous Equations**

- 1) 4,1      2) 5,3      3) 1,-1  
 4) 3,1      5) 3,-2      6) 3,5  
 7) 4,1      8) 2,3      9) 5,-3  
 10) 5,4      11) 2,-1      12) 3,-2  
 13) 5,6      14) 9,-1      15) 2.5,1  
 16) 4,-2  
 17) a)  $2x + 2y = 14$     $x + 4y = 13.60$       b)  $x = 4.80$   
 c) Adults £4.80      Children £2.20  
 18) a)  $x + y = 39$        $x - y = 9$       b) 24,15  
 19) 25.8 by 2.7      20) 90g and 120g  
 21) £1.72 and £1.02      22) 17 and 6  
 23) 5, 3      24) £22, £12  
 25) 11, 7      26) 28p and 22p

**22. Problems Involving Quadratic Equations**

- 1) a)  $x = 7$       b)  $x = 6.5$  cm  
 2) a)  $x = 10$       b)  $105 \text{ cm}^2$   
 3) a)  $\frac{20}{x} + \frac{10}{x-5} = 4$        $x = 2.5$  or 10  
 b) 10 kph and 5 kph  
 4) a)  $x^2$  and  $8x$       b)  $x^2 + 8x = 105$   
 so  $x^2 + 8x - 105 = 0$       c) 64 cm  
 5) a)  $2x + 4$  and  $x + 4$       d)  $x = 4$  m      8m by 4m.  
 6) a)  $(x+4)(x-1)$  and  $3x(x-2)$   
 b)  $x = 4$       c) 8 km/hour

**23. Straight Line Graphs****Exercise 1**

- 1) 1      2) 3      3) -5      4) -4  
 5) 2      6) 2      7)  $-\frac{5}{3}$       8)  $-\frac{1}{4}$   
 9) 2      10) 2      11) -1      12) -1  
 13) -1      14) -1      15)  $\frac{4}{3}$       16)  $\frac{1}{3}$

**Exercise 2**

- 1)  $y = x - 3$       2)  $y = 2x - 10$       3)  $y = 3x$   
 4)  $y = -x + 2$       5)  $y = -3x + 4$       6)  $y = 4x - 14$

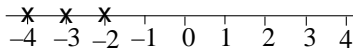
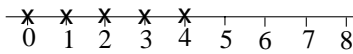
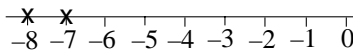
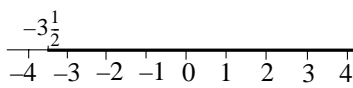
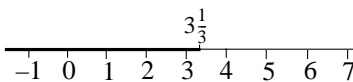
**Exercise 3**

- 1) b)  $m = 2$ ,  $c = -6$       c) 124      d) 120  
 2) b)  $m = 5$ ,  $c = 20$       c) 95      d) 17  
 3) a)  $C = 10H + 40$       b) £90      c) £105      d) 6 hours

**24. Inequalities****Exercise 1**

- 1)  $x > 6$       2)  $x > 2$       3)  $x > 8$       4)  $x > -3$   
 5)  $x > -3$       6)  $x > 2$       7)  $x > 6$       8)  $x > 4$   
 9)  $x > 5$       10)  $x < 1$       11)  $x \leq 3$       12)  $x \leq 5$   
 13)  $x \geq 8$       14)  $x < 3$       15)  $x < 5$       16)  $x > 2\frac{2}{5}$   
 17)  $x \leq 11$       18)  $x \leq -7\frac{2}{3}$       19)  $x \leq -1$       20)  $x \geq -26\frac{1}{2}$   
 21)  $x \leq -2$

**Exercise 2**

- 1)   
 2)   
 3)   
 4)   
 5) 

- 6) 2  
 7) 1  
 8) a)  $20x < 10x + 70$   
 b)  $x < 7$       Company A is cheaper when the number of hours is less than 7  
 Company A and B offer the same price when the number of hours is 7  
 Company B is cheaper when the number of hours is greater than 7

**25. Linear Inequalities**

- 1) (4,6) (5,5) (5,6) (6,4) (6,5) (6,6) (6,7)  
 2) (0,6) (1,4) (1,5) (1,6) (2,6)  
 3) (2,4) (3,3)  
 4) (3,4) (3,5)  
 5) (3,4) (3,5) (2,5)  
 6) (2,0) (3,0) (4,0) (3,-1)  
 7) (1,-1) (2,-1)  
 8) (1,2) (1,3) (2,3)  
 9) (0,2) (-1,3) (0,3) (1,3) (0,4) (0,5)



**26. Linear Programming**

- 1) a)  $y \geq 7$   $y \leq 2x$   $x + 2y \leq 20$  c) 360kg d) 6,7 and 4,8  
 2) a)  $x + y \leq 50$   $y \leq 2x$   $x \leq 2y$   
 c) i)  $33\frac{1}{3}$  tins of red,  $16\frac{2}{3}$  tins of orange  
 ii)  $33\frac{1}{3}$  tins of yellow,  $16\frac{2}{3}$  tins of orange  
 d) 8.4 litres  
 3) a) i)  $2x + y \leq 100$  ii)  $5x + 8y \geq 400$  iii)  $y \leq 2x$   
 c) 25 large, 50 small.  
 4) a) i)  $x + y < 15$  ii)  $x + y > 10$  iii)  $y > x$  iv)  $y < 2x$   
 c) (4,7) (5,6) (5,7) (5,8) (6,7) (6,8) (5,9)

**27. Recognising Graphs**

- 1) c 2) a 3) c 4) a

**28. Graphs 1**

- 1) a)

5	0	-3	-4	-3	0	5
---	---	----	----	----	---	---

d) -2.8 and 1.8

e)  $x^2 + x - 5 = 0$

- 2) a)

-8.625	-2	3	2	1	6	12.625
--------	----	---	---	---	---	--------

b)  $x = -2.25, 0$  and 2.25

- 3) a)

13	3	-3	-5	-3	3	13
----	---	----	----	----	---	----

c)  $x = -0.6$  and 2.6

d)  $x = -1.1$  and 3.1

- 4) a)

-1	0	1	2	3	4	5
-5	-4	-3	-2	-1	0	1
5	0	-3	-4	-3	0	5

d) -2.55 and 1.6  $x^2 + x - 4 = 0$

**29. Graphs 2**

- 1) a)

17	10	$8.\dot{3}$	8	8.2	$8.\dot{6}$	10	$13.\dot{3}$	15.1
----	----	-------------	---	-----	-------------	----	--------------	------

d) 0.6 and 6.4 e) 0.8 and 4.0 f) 1.5

- 2) a)

-24	-14	-6	0	4	6	6	4	0	-6
-----	-----	----	---	---	---	---	---	---	----

c)  $x = -0.25$  and 3.25

e)  $x = -1$  and 4

f)  $y = -4$ .  $x = -1.7$  or 4.7

- 3) a)  $x = -3.25$  and 1 b)  $x = -2.2$  and 0.7

**30. Graphs 3**

- 1) c)  $c = 1.5$

2) b)  $a = 3, b = 50$  c) 197

3) b)  $p = 3, q = 2$  c) 14.3

4) b)  $a = 1.5, b = -0.5$  c) 0.56

5) b)  $a = 3, b = 2$  c) 6.2

**31. Growth and Decay**

- 1) a)

0	1	2	3	4	5	6
1	0.667	0.444	0.296	0.198	0.132	0.088

c) i) 0.85 ii) 0.24

- 2) a)

0	1	2	3	4	5	6	7
10	11	12.1	13.31	14.64	16.11	17.72	19.49

c) 16.9 million d)  $4\frac{1}{4}$  years

- 3) a)

0	0.5	1	1.5	2	2.5	3	3.5	4
100	57.7	33.3	19.3	11.1	6.4	3.7	2.1	1.2

c) i) 0.63 days (15 hours 7 minutes)

ii) 8.44% iii) 0.26 days

**32. Distance - Time Diagrams**

- 1) a) 3.2 miles per hour b) 1.61

c) 1.7 miles per hour d) 2.6 miles per hour

- 2) a) 0.8 b)  $2.3ms^{-1}$

3) a)  $8.8ms^{-1}$  b) Speed is increasing (accelerating)

c) Speed is approximately constant

d) Speed is decreasing

- 4) a) 30 metres b)  $6ms^{-1}$  c) 12m d) 37.5m

**33. Velocity - Time Diagrams**

- 1) a) Area = 363 b) 363 metres c)  $2.4ms^{-2}$

2) a) 2880 metres b)  $0.5ms^{-2}$

3) 995 metres

- 4) a)

0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	3	7	12	18	25	33	42	52	63	75

b) 40.75 metres c)  $17ms^{-2}$

**34. Angles and Triangles**

1)  $49^\circ, 131^\circ, 49^\circ$  2)  $79^\circ, 84^\circ, 17^\circ$

3)  $120^\circ, 60^\circ, 30^\circ$  4)  $42^\circ, 45^\circ, 93^\circ$

5)  $60^\circ, 60^\circ, 60^\circ, 30^\circ$  6)  $57^\circ, 57^\circ, 33^\circ$

7)  $45^\circ, 27^\circ, 45^\circ$  8)  $44^\circ, 69^\circ, 67^\circ$

9)  $150^\circ, 150^\circ, 30^\circ$  10)  $40^\circ, 70^\circ, 30^\circ$

**35. Regular Polygons**

- 1) a)  $135^\circ$  b)  $67.5^\circ$  c)  $45^\circ$  2)  $360^\circ$

3)  $360^\circ$  minus the two interior angles of the shapes

4)  $x = 108^\circ$  (Interior angle of a regular pentagon)

$y = 36^\circ$  ( $180^\circ - 2 \times 72^\circ$ )

5) Regular triangle with side  $3x$

6) a)  $128.6^\circ, 77.1^\circ$  b)  $\angle HCB = \angle HDA$

7) 24 sides

**36. Congruent Triangles**

1) All 2) a and b 3) a and c 4) b and c

5) Two sides and the included angle are equal on both triangles.

- 6) Two angles and a side are equal on both triangles.  
7) (i) True - rest are false.

**37. Geometry of a Circle 1**

- 1)  $48^\circ, 42^\circ, 42^\circ$       2)  $90^\circ, 33^\circ, 57^\circ$   
3)  $140^\circ, 80^\circ, 50^\circ$       4)  $96^\circ, 84^\circ, 72^\circ$   
5)  $12^\circ, 57^\circ, 78^\circ$       6)  $27.5^\circ, 125^\circ, 97.5^\circ$

**38. Geometry of a Circle 2**

- 1)  $94^\circ, 43^\circ, 52^\circ, 42^\circ$       2)  $47^\circ, 88^\circ, 9^\circ$   
3)  $50^\circ, 130^\circ, 120^\circ, 70^\circ$       4)  $124^\circ, 56^\circ, 34^\circ$   
5)  $124^\circ, 68^\circ, 56^\circ$       6)  $62^\circ, 124^\circ, 56^\circ, 28^\circ$

**39. Vectors 1**

- 1)  $\mathbf{b} + \mathbf{c}$ ,  $\mathbf{b} + \mathbf{c} + \mathbf{d}$ ,  $\mathbf{c} + \mathbf{d}$       2)  $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$       3)  $\overrightarrow{AD}$   
4) a)  $4\mathbf{c} - 4\mathbf{a}$     b)  $2\mathbf{c} - 2\mathbf{a}$     c)  $4\mathbf{a} + 2\mathbf{c} - 2\mathbf{a} = 2\mathbf{c} + 2\mathbf{a}$   
    d)  $2\mathbf{c}$     e) PX is parallel to OC and half its length  
5) a)  $9\mathbf{b} - 6\mathbf{a}$     b)  $3\mathbf{b} - 2\mathbf{a}$     c)  $6\mathbf{a} + 3\mathbf{b} - 2\mathbf{a} = 4\mathbf{a} + 3\mathbf{b}$   
6) a)  $\mathbf{a} + \mathbf{b}$       b)  $\frac{1}{2}(\mathbf{a} + \mathbf{b})$     c)  $\frac{1}{2}(\mathbf{b} - \mathbf{a})$   
7)  $-\mathbf{a}$ ,  $-\mathbf{b}$ ,  $2\mathbf{a}$ ,  $-\mathbf{a}$ ,  $\mathbf{b} - \mathbf{a}$ ,  $2\mathbf{a} - 2\mathbf{b}$   
8) a) AB is parallel to CD, CD is twice as long as AB  
    b) Parallel. One is  $\frac{2}{3}$  the length of the other  
9) a)  $2\mathbf{b} - 2\mathbf{a}$     b)  $\mathbf{b} - \mathbf{a}$     c)  $\overrightarrow{XY}$  is parallel to  $\overrightarrow{AB}$   
    because  $\overrightarrow{AB} = 2 \times \overrightarrow{XY}$   
10)  $\overrightarrow{AC} = \mathbf{a} + \mathbf{b}$        $\overrightarrow{BD} = \mathbf{b} - \mathbf{a}$

**40. Vectors 2**

- 1) a)  $\frac{2}{3}\mathbf{b}$     b)  $\frac{1}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$     c)  $\mathbf{b} + \frac{1}{2}\mathbf{a}$   
2) a)  $\mathbf{a} + \mathbf{b}$     b)  $\frac{1}{3}(\mathbf{a} + \mathbf{b})$     c)  $\frac{1}{3}\mathbf{b}$   
    d)  $\frac{2}{3}\mathbf{a}$     PQ is parallel to OC and  $\frac{2}{3}$  of its length  
3) a)  $(\mathbf{a} + \mathbf{b})$     b)  $\frac{1}{6}(\mathbf{a} + \mathbf{b})$     c)  $\mathbf{b} - \mathbf{a}$     d)  $\frac{1}{6}(\mathbf{b} - \mathbf{a})$   
    e)  $\frac{2}{3}\mathbf{a}$     f) They are parallel and XY is  $\frac{2}{3}$  the length of CB  
4) a)  $\frac{1}{2}\mathbf{a}$     b)  $\mathbf{b} - \frac{1}{2}\mathbf{a}$     c)  $\mathbf{a} + \mathbf{b}$     d)  $\frac{1}{3}\mathbf{b} - \frac{1}{6}\mathbf{a}$   
    e)  $\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$     f)  $\frac{1}{3}$   
5) a)  $12\mathbf{b} + 6\mathbf{a}$     b)  $5\mathbf{a} - 2\mathbf{b}$     c)  $3\mathbf{b} + 6\mathbf{a}$

**41. Similar Shapes**

- 1) a) 4:9    b) 8:27    c)  $337.5 \text{ cm}^3$   
2) a) 8:19    b)  $450 \text{ cm}^2$   
3) 122.88 grams    4) 390.625 grams  
5) 11.9 cm    6)  $8,000 \text{ cm}^3$   
7) a)  $112.5 \text{ cm}^2$     b)  $14.81 \text{ cm}^3$   
8) 350 grams

**42. Similarity**

- 1) a) i)  $75^\circ$     ii) Since DC and BE are parallel then angle CDE and BEA are equal  
    b) AE = 6 cm  
2) a) i)  $39^\circ$     ii) Angle DCE =  $116^\circ$  (vertically opposite) Angle DEC =  $180 - (116 + 25) = 39^\circ$   
    (3 angles of a triangle add up to  $180^\circ$ )  
    b) 7.5 cm  
3) a) 7.5 cm    b) 7.5cm    4) a) 6 cm 1.5cm    b) 1:16

**43. Bearings**

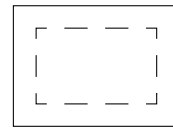
- 1) S  $28^\circ$  W ( $208^\circ$ )      2) N  $73^\circ$  E ( $073^\circ$ )  
3) a) N  $30^\circ$  E ( $030^\circ$ )      b) S  $30^\circ$  E ( $150^\circ$ )  
    c) N  $30^\circ$  W ( $330^\circ$ )  
4) a) N  $6^\circ$  E ( $006^\circ$ )      b) N  $55^\circ$  W ( $305^\circ$ )  
    c) S  $55^\circ$  E ( $125^\circ$ )  
5) a) N  $60^\circ$  E ( $060^\circ$ )      b) Due South ( $180^\circ$ )  
    c) S  $60^\circ$  W ( $240^\circ$ )      d) S  $60^\circ$  W ( $240^\circ$ )  
6) a) N  $51^\circ$  W ( $309^\circ$ )      b) N  $4^\circ$  E ( $004^\circ$ )  
    c) N  $84^\circ$  W ( $276^\circ$ )

**44. Construction**

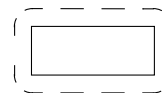
- 6) 21 metres    7) 4.5 cm    8) 1630 m

**45. Loci**

1)

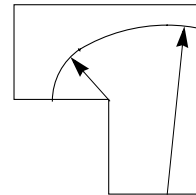


2)



3) 81 metres

4)



5) 700 metres

**46. Transformations 1**

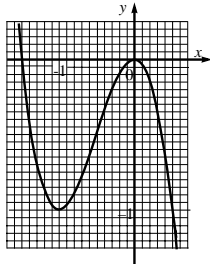
- 1) a)  $(-1, -2)$   $(-3, -2)$   $(-4, -5)$   
    b)  $(1, -2)$   $(3, -2)$   $(4, -5)$   
    c) Rotation of  $180^\circ$  about origin  $(0,0)$   
2) a)  $(2,2)$   $(2,4)$   $(4,2)$   
    b)  $(2,-2)$   $(2,-4)$   $(4,-2)$   
    c)  $(-1,1)$   $(-2,1)$   $(-1,2)$   
    d) An enlargement with a scale factor of 2 through the origin  $(0,0)$   
3) b)  $(1,2)$   $(2,2)$   $(2,1)$   
    c) An enlargement with a scale factor of  $2\frac{1}{2}$  through the origin  $(0,0)$

**47. Transformations 2**

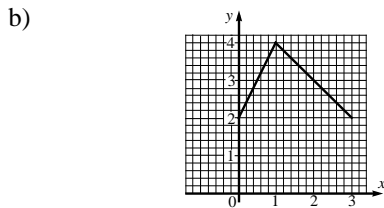
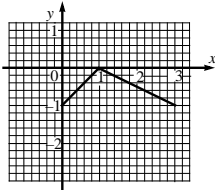
- 1) a)  $(-1,-1)$   $(-4,-4)$   $(-3,-1)$   
    b)  $(-1,3)$   $(-3,3)$   $(-4,0)$   
    c) Rotation of  $180^\circ$  (clockwise or anticlockwise) about the point  $(0,2)$   
2) a) Rotation of  $180^\circ$  about the origin  $(0,0)$   
    b) Translation  $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$     c)  $(-1,4)$   $(-1,2)$   $(1,2)$   
    d) Rotation of  $180^\circ$  about  $(-1,3)$   
3) a)  $(2,1)$   $(5,1)$   $(4,-2)$     b)  $(0,1)$   $(0,-2)$   $(-3,-1)$   
    c) Reflection in the line  $y = 3 - x$

**48. Transformations of Graphs**

- 1) b) iii c) v d) iv e) ii f) i  
 2) b) Translation  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  c) Translation  $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$   
 d) y dimensions  $\times 4$  e) Translation of  $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$   
 f) Translation of  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$   
 3) a)  $-2x^3 - 3x^2$   
 b)



- c)  $-2x^3 + 3x^2$ . Reflection in the y axis.  
 4) a)



**49. Matrix Transformations**

- 1) a) Rotation of  $180^\circ$   
 b) Enlargement of  $\times 2$  through origin (0,0)  
 c) Reflection in the y axis  
 d) Reflection in the line  $y = x$   
 2)  $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$  (1,2)  
 3) a) Points are (-1,1) (-2,1) (-1,3) and (1,-1) (2,-1) (1,-3)  
 b)  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  c)  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$   
 d)  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  Reflection in the x axis ( $y = 0$ )  
 4) a)  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$  b) Rotation of  $90^\circ$  anticlockwise about (0,0)  
 c) Rotation of  $90^\circ$  clockwise about (0,0)  $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$   
 5) a) (1,-1) (2,-2) (1,-4) (4,-1)

- b) Reflection in the x axis ( $y = 0$ )

d)  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$  (5,3)

**50. Area and Perimeter**

- 1)  $60 \text{ cm}^2$  2)  $35.3475 \text{ cm}^2$   
 3) a)  $9.426 \text{ cm}^2$  3.142 cm  
 b)  $175.952 \text{ cm}^2$  29.3 cm (to 1 d.p.)  
 c)  $475.2 \text{ cm}^2$  (to 1 d.p.) 57.6 cm (to 1 d.p.)  
 d)  $477.5 \text{ cm}^2$  (to 1 d.p.) 57.0 cm (to 1 d.p.)  
 4) a)  $42.55 \text{ m}^2$  (to 2 d.p.) b) 27.02 m (to 2 d.p.)  
 5) a)  $333.8 \text{ cm}^2$  (to 2 d.p.) b) 74.2 cm (to 1 d.p.)  
 6) a)  $62.84 \text{ m}^2$  b) 14 tins 7)  $402.2 \text{ m}^2$  (to 1 d.p.)  
 8)  $509,500,000 \text{ km}^2$  (to 4 sig. figures)

**51. Volume**

- 1)  $960 \text{ cm}^3$  2) 7.5 cm  
 3) a)  $752.8 \text{ cm}^3$  (to 4 sig. figures)  
 b) 564.6 grammes (to 4 sig. figures)  
 4) a)  $134.06 \text{ cm}^3$  b) 18.1 kg  
 5) 4.63 cm (to 2 d.p.) 6) 330,000 litres  
 7) 16 litres 8) 5893 spheres  
 9) a)  $98 \text{ cm}^3$  b) 20  
 10) a)  $5832 \text{ cm}^3$  b) 52.4%  
 11) 3.4 cm (to 1 d.p.)  
 12)  $255.55 \text{ cm}^3$  (to 2 d.p.) 13) 156 ml

**52. Ratios and Scales**

- 1) 120, 150, 180 2) 150, 175  
 3) a) 2:4:1 b) 160, 320, 80  
 4) 450 ml 5) 50 cm, 2.2 m,  $500 \text{ cm}^2$ ,  $40^\circ$   
 6) 100 cm,  $80^\circ$ ,  $120 \text{ cm}^2$ , 12.5 m  
 7) 150 cm,  $7.68 \text{ m}^2$ ,  $93,750 \text{ cm}^3$  10 cm table  
 8) a) 50 cm b)  $80 \text{ m}^2$  c)  $3,750 \text{ cm}^3$

**53. Degree of Accuracy**

- 1) 84, 86 2) a)  $418.65 \text{ cm}^3$  b)  $403.27 \text{ cm}^3$   
 3) 368 4) 0.35 cm, 0.05 cm  
 5) a)  $9.79 \text{ cm}^3$  b)  $8.24 \text{ cm}^3$  c) 15.8%  
 6) a) 89.22 litres b) 5.07 litres  
 7) a) 22 b) 20 c) 440 d) 20

**54. Formulae**

**Exercise 1**

- |            |            |            |
|------------|------------|------------|
| 1) area    | 2) area    | 3) area    |
| 4) none    | 5) area    | 6) area    |
| 7) length  | 8) length  | 9) length  |
| 10) volume | 11) volume | 12) volume |
| 13) none   | 14) none   | 15) length |
| 16) length | 17) area   | 18) volume |
| 19) none   | 20) area   | 21) volume |

**Exercise 2**

- 1) a) iii    b) i    2) a) ii    b) v

**55. Pythagoras Theorem**

All to 4 significant figures

- 1) a) 7.211    b) 7.483    c) 4.8    d) 8.801  
 2) 45.5 cm    3) 5.657 cm, 2.828 cm  
 4) 6.403 cm    5) 2.721 metres    6) 18.81 metres

**56. Sine, Cosine and Tangent Ratios**

- 1) (to 4 sig. figures) a) 15.28 cm    b) 6.576 cm  
 c) 5.503 cm    d) 22.58 cm  
 2) a)  $41.2^\circ$     b)  $46.9^\circ$     c)  $42.3^\circ$     d)  $42.2^\circ$   
 3) CD = 17.33 cm    AC = 26.69 cm  
 EB = 10.20 cm    DEB =  $43.3^\circ$   
 4) a) N $24^\circ$ E ( $024^\circ$ )    b) 5.41 km (to 2 d.p.)  
 5) a) 7.523 metres (to 4 sig. figures)    b)  $28^\circ$

**57. Sine and Cosine Rules**

- 1) a)  $72.97^\circ$     b)  $61.09^\circ$     c)  $4.74^\circ$     d)  $54.79^\circ$   
 e)  $54.80^\circ$     f) 10.54 cm  
 2) 16.01 cm    3)  $115.18^\circ$  and  $64.82^\circ$   
 4) 4.05 km    5) 16.08 m  
 6)  $58.75^\circ$ ,  $71.79^\circ$ ,  $49.46^\circ$

**58. Areas of Triangles**

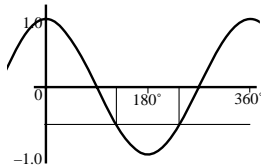
- 1)  $20.99 \text{ cm}^2$     2)  $53.83 \text{ cm}^2$     3)  $22.36 \text{ cm}^2$   
 4)  $27.39 \text{ cm}^2$     5)  $131.95 \text{ cm}^2$   
 6) The perpendicular height from C to AD is  $9 \sin 20^\circ$ .  
 The perpendicular height from B to DA produced is also  $9 \sin 20^\circ$ . Triangles ACD and ABD have the same base ie. AD. Therefore their areas are equal.  
 7)  $49.85 \text{ cm}^2$     8)  $2.3 \text{ m}^2$   
 9)  $237.8 \text{ cm}^2$     10) 3.4 hectares

**59. Trigonometry Mixed Exercise**

- 1) AC = 6.325 cm    CD = 5.175 cm  
 2) 10.91 cm    3) 9.226 m  
 4)  $122.75^\circ$ ,  $114.5^\circ$     5) 14.62 cm    6) 13.09 m

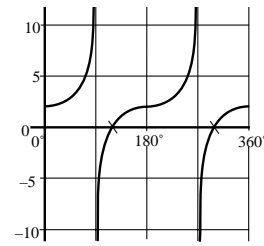
**60. Graphs of Sines, Cosines and Tangents**

- 1)  $210^\circ$  and  $330^\circ$     2) Approx  $49^\circ$  and  $131^\circ$   
 3)

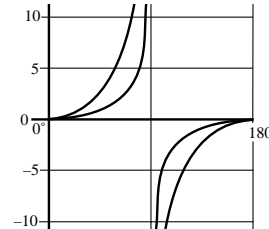


- 4) Approx  $71^\circ$  and  $289^\circ$     5) Approx  $76^\circ$  and  $256^\circ$

- 6) a)



- b)



- 7) Approx
- $56^\circ$
- and
- $236^\circ$

**61. Three Dimensional Trigonometry**

- 1) 33.31 m (to 4 sig. figures)  
 2) a) 28.28 cm    b) 20.20 cm  
 3) a)  $4.667 \text{ m}$     b)  $23.2^\circ$     c) HC = 5.077 m  
 4) a) 3.830 m    b)  $21.0^\circ$   
 5) 107.3 m

**62. Questionnaires**

The following answers are for guidance only. There are other solutions.

- 1) a) The survey could be biased for the following reasons:  
 i) It is done just in the month of July only.  
 ii) It is carried out at 12.00 only and not at other times of the day  
 iii) It is carried out in just one particular place. To get information about the whole of Britain she needs information from other towns.  
 iv) Town is of one particular type ie historical.  
 b) The survey could be improved by including the points raised in part a)  
 c) Are you a tourist ?  
 Which country do you come from ?  
 What is your gender m/f ?  
 2) a) How old are you ?  
 What is your gender, male or female ?  
 What other newspapers do you read?  
 b) i) It only asks those who already read the newspaper. It does not get to potential readers.  
 3) a) It tells her the age of the cars on the road now. She could, for example, calculate the average age of cars on the road but not the age at which they are taken off the road.  
 b) i) Was your last car sold to someone to use again or was it scrapped ?  
 ii) How old was the car when it was scrapped ?

**63. Sampling**

- 1) a) 14, 12, 14, 12, 12, 4, 2  
 b) Do you watch TV ? Yes/no  
 What is your gender ? Male/female  
 Which channels do you watch mostly ? 1, 2, 3 etc.  
 Which types of programmes do you mainly watch ? Plays, sport, news etc.
- 2) a) 10, 14, 10, 4, 2  
 b) Are you happy with your working conditions ?  
 What areas of your working environment need to be improved?  
 What type of work do you do ?  
 How old are you ?  
 Gender, male/female.
- c) Age range, type of work undertaken, gender etc.
- 3) a) 19, 37, 33, 11  
 b) i) What would you like to see improved in the centre ?  
 ii) Which facilities do you use most often?  
 iii) Would you use other facilities if they were introduced ? Yes/no  
 iv) What other facilities do you think ought to be introduced ? (give a choice)

**64. Scatter Diagrams**

- 1) b) 235  
 2) a) 7:30 pm      b) 3°C      c) 4:45 pm  
 3) a) 15.5 tonnes (approximately)      b) 4.8 minutes

**65. Pie Charts**

- 1) Business stock £5972    Employee costs £14722  
 Premises £7361    General administration £10278  
 Advertising £3889    Other costs £7778
- 2) Angles 115.2°, 43.65°, 37.35°, 59.4°, 83.25°, 21.15°
- 3) a) 1008    b) 140°    d) 126

**66. Flow Charts 1**

- 1)  $n$        $S$   
 40      1603  
 41      1684  
 42      1767  
 43      1852  
 44      1939  
 45      2028
- There are 44 numbers less than 2000
- 2) W. Jones - Merit  
 J. Connah - Pass  
 C. Smith - Distinction  
 H. Patel - Pass

**67. Flow Charts 2**

- 1) 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181.  
 Each number is obtained by adding together the previous two numbers.
- 2)  $M = 8$  It calculates the mean of a list of numbers.

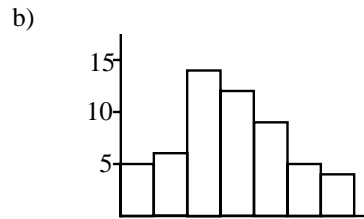
**68. Histograms 1**

1) a)

Time	0–	5–	10–	15–	20–	25–	30–	35–	40–	45–50
Freq	7	12	22	28	33	24	14	8	6	3

- b) mean = 21.83 minutes  
 c) 35%  
 2) a)

Mass	40–	50–	60–	70–	80–	90–	100–110
Freq.	5	6	14	12	9	5	4



c) 60–70

**69. Histograms 2**

1) a)

Age	15–	25–	30–	35–	45–	60–70
People	18	21	28	32	12	6

b) mean = 35.45  
 2) a)

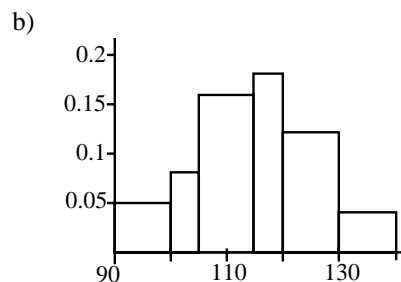
Speed	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 100$
No. of cars	36	80	208	112	60

b)  $40 < x \leq 60$  is the modal range

**70. Histograms 3**

1) a)

Height in cm	No. of plants	Frequency density
$90 < x \leq 100$	5	0.5
$100 < x \leq 105$	4	0.8
$105 < x \leq 115$	16	1.6
$115 < x \leq 120$	9	1.8
$120 < x \leq 130$	12	1.2
$130 < x \leq 140$	4	0.4

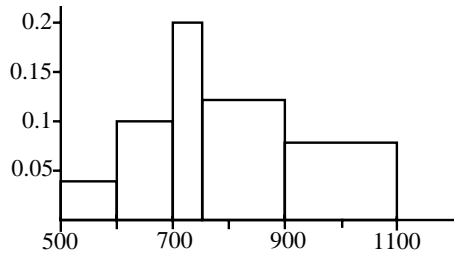


c) 50%

2) a)

Time	Frequency	Frequency density
$500 < x \leq 600$	4	0.04
$600 < x \leq 700$	10	0.10
$700 < x \leq 750$	10	0.20
$750 < x \leq 900$	18	0.12
$900 < x \leq 1100$	16	0.08

b)



c) 52%

**71. Mean**

1)

Wages, £	Frequency	Mid value	Frequency x mid value
60 - 100	4	80	320
100 - 140	19	120	2280
140 - 170	24	155	3720
170 - 200	11	185	2035
200 - 220	6	210	1260

mean = £150.23

2)

Weight	Frequency	Mid value	Frequency x mid value
50 - 70	5	60	300
70 - 90	14	80	1120
90 - 100	16	95	1520
100 - 110	24	105	2520
110 - 120	21	115	2415
120 - 140	23	130	2990
140 - 160	17	150	2550

mean = 111.8 cm

3)

Speed	Frequency	Mid value	Frequency x mid value
$20 \leq s < 30$	6	25	150
$30 \leq s < 40$	10	35	350
$40 \leq s < 50$	15	45	675
$50 \leq s < 60$	14	55	770
$60 \leq s < 70$	6	65	390
$70 \leq s < 80$	6	75	450
$80 \leq s < 90$	3	85	255
Totals	60		3040

mean = 50.67

**72. Mean, Median and Mode**

- 1) a) 21 pints    b) 18.3 pints    c) 18 pints  
 2) a) 4.3 letters    b) 3    c) 2  
 d) No - because the sample is very small compared with the whole book and it is only taken from one part of the book.  
 3) b) 428.3 kg    c) 100 - 300    d) 100 - 300  
 4) a) 1.02 - 1.03 litres    b) 1.02 - 1.03 litres  
 c) 1.025 ml (to 4 sig. figures)

**73. Mean and Standard Deviation**

- 1) a) 5.044, 0.0246  
 b) i) The mean is increased by 0.02 kg  
 ii) There is no change to the standard deviation  
 2) b) i) 155.6    ii) 10.7  
 3) b) £230.15, 67.48    c) Mean is increased by £10, standard deviation remains the same.  
 4) b) 41 and 12.54  
 c) The mean age at the badminton club is higher, indicating that the members tend to be older. The S.D. at the badminton club is lower, indicating that a greater proportion of the ages are closer to the mean than at the tennis club.

**74. Frequency Polygons**

1) a)

Weight	Frequency	Mid value
3.0 - 3.5	4	3.25
3.5 - 4.0	6	3.75
4.0 - 4.5	12	4.25
4.5 - 5.0	5	4.75
5.0 - 5.5	3	5.25

2)

Height	Frequency	Mid value
80 - 100	8	90
100 - 120	14	110
120 - 140	26	130
140 - 160	34	150
160 - 180	18	170
180 - 200	8	190
200 - 220	4	210

**75. Cumulative Frequency 1**

1) a)

Length <i>l</i>	9.7	9.8	9.9	10.0	10.1	10.2
Cumulative Frequency	4	17	41	77	94	100

c) 9.92 cm    d) 9.99, 9.83, 0.16    e) 10%

2) a)

Age	20	30	40	50	60	65
Cumulative Frequency	17	82	160	211	241	250

c) 26%

3) a)

Mark	10	20	30	40	50	60	70	80	90	100
Cumulative Frequency	3	10	32	63	97	134	162	181	194	200

c) 44    d) 14%

**76. Cumulative Frequency 2**

1) a)

Life of bulb	6000	7000	8000	9000	10,000	11,000
Cumulative Frequency	5	25	67	162	242	300

c) 86%

2) a)

Time spent in air	30	35	40	45	50	55
Cumulative Frequency	6	16	28	62	85	100

c) 43 minutes, 10 minutes

3) a)

Timing (minutes)	5 early	0	5 late	10 late	15 late	20 late
Cumulative Frequency	33	99	121	138	148	150

c) 15%

4) a)

No. of T.V.'s	100	150	200	250	300	350
Cumulative Frequency	5	32	109	254	337	365

c) 246 days    d) 10 days

**77. Probability 1**

- 1) a)  $\frac{5}{9}$     b)  $\frac{20}{81}$     c)  $\frac{125}{729}$     d)  $\frac{80}{243}$   
 2) a)  $\frac{1}{6}$     b)  $\frac{1}{4}$     c)  $\frac{5}{6}$   
 3) a) 0.16    b) 0.48    c) 0.36  
 4) a) 0.02    b) 0.02    c) 0.01  
 5) a) 0.14    b) 0.06    c) 0.24    d) 0.86  
 6) a) 0    b)  $\frac{5}{9}$

**78. Probability 2**

- 1) a)  $\frac{3}{10}$     b)  $\frac{7}{30}$     c)  $\frac{7}{40}$     2) 0.96    3) 0.996625  
 4) a)  $\frac{1}{2}$     b)  $\frac{1}{4}$     5) a)  $\frac{8}{15}$     b)  $\frac{8}{29}$     6) a)  $\frac{2}{5}$     b)  $\frac{3}{20}$   
 7) a) 0.02682    b) 0.045811    c) 0.927369

**79. Relative Probability**

- 1) 620  
 2) a) i) 0.44    ii) 0.34    iii) 0.22  
       b) 19,800; 15,300; 9,900  
 3) a) 25, 15, 10    b) i) 0.5    ii) 0.3    iii) 0.2  
       c) i) 0.6    ii) 0.4    d) 250,000 and 150,000  
 4) a) 0.25, 0.3, 0.45    b) 137, 165, 247

5) a)

No. of people	50	100	150	200	250	300
No. 'for'	30	45	55	83	96	122
No. 'against'	20	55	95	117	154	178
Probability 'for'	0.60	0.45	0.37	0.415	0.384	0.407
Probability 'against'	0.40	0.55	0.63	0.585	0.616	0.593

c) 0.4    d) 42,000

**80. Tree Diagrams**

- 1) a) 0.015, 0.085, 0.135, 0.765  
       b) 0.235    c) 0.765  
 2) a) 0.035, 0.665, 0.045, 0.255  
       b) 0.08  
 3) a)  $\frac{3}{28}$ ,  $\frac{15}{56}$ ,  $\frac{15}{56}$ ,  $\frac{5}{14}$     b) i)  $\frac{15}{28}$     ii)  $\frac{9}{14}$   
 4) a)  $\frac{3}{14}$     b)  $\frac{11}{14}$     5) a)  $\frac{20}{63}$     b)  $\frac{11}{63}$     c)  $\frac{32}{63}$

**81 Perpendicular Lines**

- 1) a) 3    b)  $-\frac{1}{3}$     c) (i), (iii), (iv), (vii).  
 2) a)  $y = x$     b)  $y = -x + 12$   
 3) a)  $y = \frac{1}{2}x + 2$     b) -2    c) (0,12)    d)  $y = -2x + 12$   
 4)  $y = -3x + 20$   
 5)  $\frac{2}{3}$     b)  $y = \frac{2}{3}x + \frac{2}{3}$  or  $y = \frac{2}{3}(x + 1)$     c) AD is  $-\frac{3}{2}$

BC is  $-\frac{3}{2}$  DC is  $\frac{2}{3}$  d) AD is  $y = -\frac{3}{2}x + 5$

BC is  $y = -\frac{3}{2}x + \frac{23}{2}$  DC is  $y = \frac{2}{3}x + 5$

**82 Completing the Square**

1) b) 4 c)  $\frac{9}{4}$  d)  $\frac{49}{4}$  e)  $\frac{1}{4}$  f)  $\frac{81}{4}$  h)  $\frac{9}{16}$  i)  $\frac{4}{7}$  j)  $\frac{1}{6}$  k)  $\frac{1}{4}$  l)  $\frac{25}{64}$

2) a) 10 or -2 b) 7 or -3 c) -2 or -9 d) -3 or -2

e) 2 or -1 f) 1 or -4 g) 5 or -1 h) 4 or -7

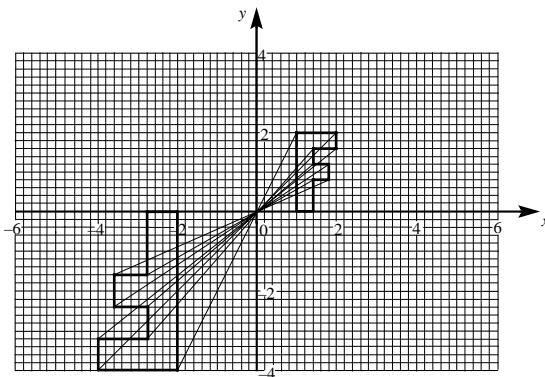
i) -1 or -3 j)  $-\frac{1}{2}$  or -2 k)  $-\frac{3}{2}$  or 2 l)  $\frac{1}{2}$  or -3

m)  $\frac{1}{4}$  or  $-\frac{1}{2}$  n)  $\frac{1}{2}$  or  $-\frac{2}{3}$  o)  $\frac{1}{3}$  or  $-\frac{3}{5}$

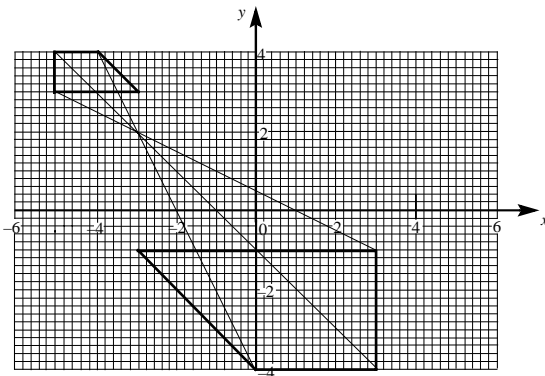
**83 Enlargement 1**

1)  $\frac{3}{2}$ ,  $\frac{1}{2}$  and -1

2)

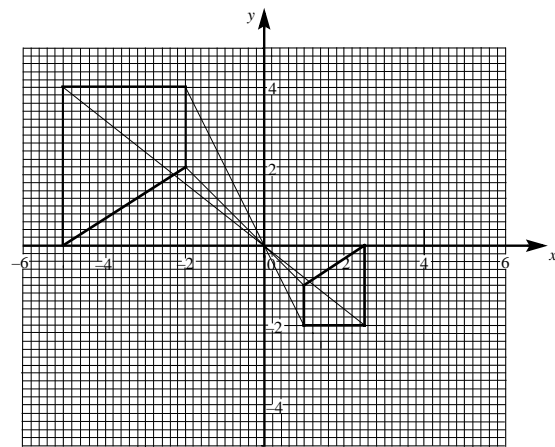


3)

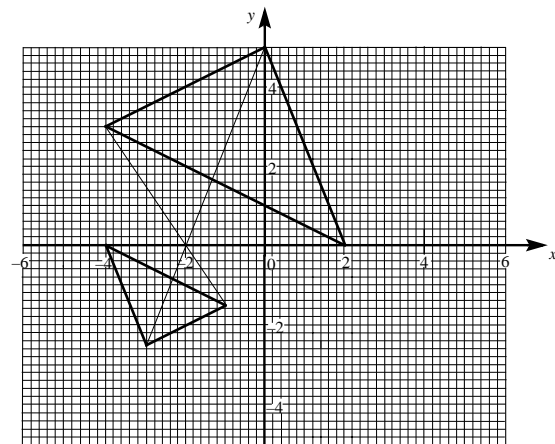


**84 Enlargement 2**

1)



2)



**85 Equation of a circle**

1) 10 2) a) A b) 6 c)  $y = x$  and  $y = x - 6$  d)  $(3\sqrt{2}, 3\sqrt{2})$  and  $(3\sqrt{2}, 3\sqrt{2})$

3) (6,8) and  $(-\frac{14}{5}, -\frac{48}{5})$  4)  $(\sqrt{51}, -7)$  and  $(-\sqrt{51}, -7)$

b)  $(\sqrt{51}, 7)$   $(-\sqrt{51}, 7)$

5) a)  $\frac{\sqrt{181}}{2}$  b) (6.6, 1.3)

**86 Simultaneous equations 2**

1) (2,3) (3,2) 2) (-5, -4) (4,5) 3) (2,5) (5,2)

4) (7.8, 0.4) (5,6) 5) (-4.8, 1.4) (-4, 3)

**87 Sampling 1**

1) 6, 29, 14, 8, 33, 10 2) a) 369, 31, 623 b) 3000, 130

3) 17, 13, 9, 6, 3, 2

**88 Sampling 2**

1) a) To get two digits you would need two dice.  
Preferably of different colours but one



distinguishable from the other. One would be designated for the first digit and the other the second digit. Throw the dice and write down the numbers.

- b) Use the same method with three dice.
- 2) a) Allocate each of the numbers 1 to 77 to each of the 77 people. Now go through the table picking off the first 10 numbers with values of 77 or less. These numbers would indicate the 10 chosen males.
- b) 11, 55, 59, 38, 33, 25, 34, 14, 74, 39
- c) The largest 2 digit number would be 99. Since there are 265 people in this group you would need to use a table containing 3 digits.
- d) A table of 3 digit numbers.

### 89 3 Dimensional Co-ordinates 1

- 1) (4,6,8) 2) a) (0,0,1) (1,0,1) (0,1,0) (1,1,0) (1,1,1) (0,1,1) b) (i) (2,2,2) (3,2,2) (3,2,3) (2,2,3) (2,3,3) (2,3,2) (3,3,2) (3,3,3) (ii) (2.5, 2.5, 2.5)
- 3) (0, 2, 0) (2, 0, 0) (2, 0, 2) (2, 2, 0)
- 4) a) (3, 3, 2) (5, 3, 4) (1, 3, 4) (3, 1, 4) (3, 5, 4) (3, 3, 6) b) (3, 3, 4)
- 5) a) (0, 0, 3) (0, 4, 0) (5, 0, 3) (5, 4, 3) b) (2.5, 0, 1.5) (5, 2, 1.5) (2.5, 4, 1.5) (0, 2, 1.5) (2.5, 2, 0) (2.5, 2, 3)

### 90 3 Dimensional Co-ordinates 2

- 1) (1, 0, 2)
- 2) a) (1, -1, 2) (5, -1, 2) (5, 3, -2) (5, 3, 2) (1, 3, 2) (1, 3, -2) b) (i) (3, 1, 0) (7, 1, 0) (3, 1, 4) (7, 1, 4) (7, 5, 0) (7, 5, 4) (3, 5, 4) (3, 5, 0) (ii) (5, 3, 2)
- 3) (-3,0,-1) (-1,-2,1) (-1,0,-1) (-1,-2,-1)
- 4) a) 6 units b) (1, -3, 1) (-2, 0, 1) (1, 3, 1) (4, 0, 1) (1, 0, -2) (1, 0, 4) c) (1, 0, 1) d) (6, 6, 0)
- 5) a) (-7, -3, -3) (-3, -6, -3) (-3, -6, -5) (-3, -3, -5) b) (-5, -6, -4) (-7, -4.5, -4) (5, -3, -4) (-3, -4.5, -4) (-5, -4.5, -3) (-5, -4.5, -5) c) (-5, -4.5, -4) d)  $2 \times 3 \times 4$  e) (4, 3, 2)

### 91 Moving Averages

- a) There needs to be at least 8 weeks data to calculate

an 8 week moving average.

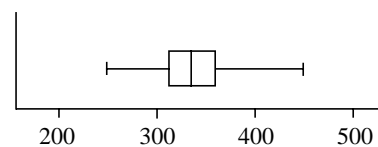
- b) At the end of week 8  
c)

Day	Value of a share in pence	8 week moving average
1	251	
2	263	
3	294	
4	330	
5	350	
6	345	
7	342	
8	340	314.4
9	336	325.0
10	331	333.5
11	330	338.0
12	321	336.9
13	315	332.5
14	302	327.1
15	321	324.5
16	317	321.6
17	320	319.6
18	318	318.0
19	307	315.1
20	301	312.6
21	331	314.6
22	287	312.8
23	305	310.8
24	306	309.4

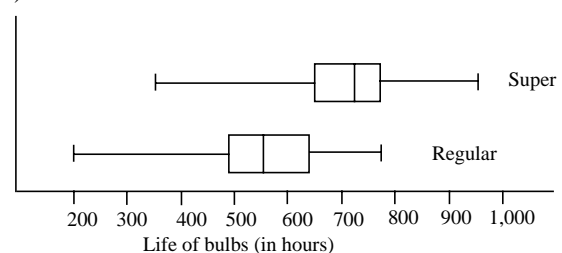
- d) The trend is down

### 92 Box Plots

1)



2)



The Super bulbs last longer in general than the Regular bulbs. However there is an overlap, with some Regular bulbs lasting longer than the Super ones.

3) Machine A. Both the upper and lower quartiles lie outside the acceptable range indicating that less than 50% of the rods are acceptable.

Machine B. Both the upper and lower quartiles lie within the acceptable range indicating that more than

50% of the rods are acceptable. Also, the largest diameter rod is smaller than the largest rod from machine A and the smallest diameter rod is larger than the smallest from machine A. All this indicates that machine B is the more accurate.