

Answers to examination-style questions

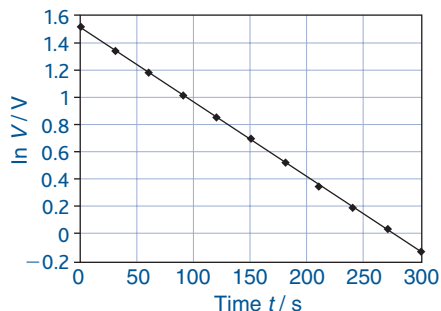
Answers	Marks	Examiner's tips
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- 1 (a) Taking natural logs on both sides of  $V = V_0 e^{-t/CR}$  gives  $\ln V = \ln V_0 + \ln (e^{-t/CR})$  **1**  
 As  $\ln (e^{-t/CR}) = -\frac{t}{CR}$  **1**  
 then  $\ln V = \ln V_0 - \frac{t}{CR} = a - bt$   
 hence  $a = \ln V_0$  **1**  
 and  $b = \frac{1}{CR}$  **1**

- (b) (i) **2** 1 mark for each correct row

$t/s$	210	240	270	300
mean $V/V$	1.427	1.233	1.033	-0.887
$\ln V$	0.356	0.209	0.032	-0.120

- (ii) **4**



- For correct labels on each axis
- for suitable scales
- for correctly plotted points
- for best-fit line.

- (iii) Time constant ( $= RC$ ) **1**

$$= \frac{1}{\text{gradient of graph}}$$

Gradient of graph  $= 5.40 \times 10^{-3} \text{ s}^{-1}$  **1**

$\therefore$  time constant  $= \frac{1}{5.40 \times 10^{-3}} = 185 \text{ s}$  **1**  
 or 190 s

$$C = \frac{\text{time constant}}{R} = \frac{185}{6.8 \times 10^4}$$
 **1**

$$= 2.72 \times 10^{-3} \text{ F}$$

$$= 2720 \text{ } \mu\text{F} \text{ or } 2700 \text{ } \mu\text{F}$$

- (c) (i) The range of each set of readings is no more than 0.03 V except for the reading at  $t = 150 \text{ s}$  which is 0.12 V. This exception is probably due to an anomalous reading. **1** A reliable reading is one that has a consistent value each time the reading is made.
- The readings are therefore reliable because the range of each set of readings is very small compared with the mean value. **1**

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<p>(ii) Apart from the exception at <math>t = 150</math> s, the uncertainty of the readings is therefore no more than <math>\pm 0.02</math> V.</p> <p>or:</p> <p>At <math>t = 0</math> the values of <math>\ln V_0</math> are 1.504, 1.506 and 1.504; a mean of 1.505 and an uncertainty of 0.001 (half the range).</p> <p>Using the values at 300 s as an example, the values of <math>\ln V</math> are <math>-0.117</math>, <math>-0.105</math> and <math>-0.139</math>; a mean of <math>-0.120</math> and an uncertainty of 0.017 (half the range).</p> <p>In finding the gradient the change in <math>\ln V</math> is divided by the time taken. The uncertainty in the change in <math>\ln V</math> from <math>t = 0</math> to 300 s is about <math>0.017 + 0.001 = 0.018</math> and as a percentage</p> $\frac{0.018}{1.504 + 0.120} \times 100 = 1.3\%$ <p>The equation for <math>C</math> is</p> $C = \frac{t}{R(\ln V_0 - \ln V)}$ <p>The error in <math>t</math> is not known, but taking the largest value of <math>t = 300</math> s, this error is likely to be small (if timing was by hand then the uncertainty in <math>t</math> will be about 0.3 s or 0.1 %) given <math>R</math> is accurate to 1%, the value of <math>C</math> is accurate to within 2.3% (<math>= 1.3\% + 1\%</math>) or about 2%.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>4 marks max for part (c)</p> <p><b>Note</b> If time permits, you could estimate the random error in <math>V</math> and hence in <math>\ln V</math> for every point and represent them as error bars on the graph. This would allow you to draw lines of maximum and minimum gradient and so determine maximum and minimum values for the time constant to give an uncertainty value for <math>C</math>.</p>
<p>2 (a) Charge on capacitor <math>Q = CV</math></p> $= 2.0 \times 10^{-6} \times 150$ $= 3.0 \times 10^{-4} \text{ C (300 } \mu\text{C)}$	<p>1</p> <p>1</p>	<p>The upper plate loses electrons and the lower plate gains an equal number of electrons. The charge on the upper plate is <math>+300 \mu\text{C}</math> and that on the lower plate is <math>-300 \mu\text{C}</math>.</p>
<p>(b) (i) The required graph is of pd <math>V</math> on the vertical axis and charge <math>Q</math> on the horizontal axis.</p>	<p>1</p>	<p>See Question 1 (above). For the energy stored to be calculated from the area <b>under</b> the line on the graph, the axes have to be this way round. The derivation using <math>\Delta W = V \Delta Q</math> requires <math>\Delta Q</math> to be on the horizontal axis.</p>
<p>(ii) Energy stored <math>= \frac{1}{2}CV^2</math></p> $= \frac{1}{2} \times 2.0 \times 10^{-6} \times 150^2$ $= 2.25 \times 10^{-2} \text{ J (22.5 or 23 mJ)}$	<p>1</p> <p>1</p>	<p>You could use <math>\frac{1}{2}QV</math>, where <math>Q</math> is the <math>300 \mu\text{C}</math> determined in part (a). When finding energy stored, <math>\frac{1}{2}CV^2</math> is usually the safest approach because both <math>Q</math> and <math>V</math> change as a capacitor charges, but <math>C</math> is constant as <math>V</math> changes.</p>

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<p>(c) (i) Maximum discharge current  <math display="block">= \frac{V}{R} = \frac{150}{220 \times 10^3} = 6.82 \times 10^{-4} \text{ A}</math>                     or <math>6.8 \times 10^{-4} \text{ A}</math></p> <p>(ii) Since <math>Q = Q_0 e^{-t/RC}</math>, and <math>Q \propto V</math>,                      it follows that <math>V = V_0 e^{-t/RC}</math> and that  <math>I = I_0 e^{-t/RC}</math> (current <math>I \propto V</math>)                      Time constant of circuit = <math>RC</math>  <math>= 220 \times 10^3 \times 2.0 \times 10^{-6} = 0.44 \text{ s}</math>                      When <math>I = 0.1 I_0</math>, substitution gives  <math>0.1 I_0 = I_0 e^{-t/0.44}</math>                      or <math>e^{-t/0.44} = 0.1</math>  <math>\therefore e^{t/0.44} = \frac{1}{0.1} = 10</math>                      taking logs, <math>\frac{t}{0.44} = \ln 10</math>                      giving time <math>t = 1.01 \text{ s}</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>The maximum current is at the start, as soon as the resistor is connected across the capacitor. The pd across the capacitor is 150 V as it starts to discharge.</p> <p>The quantitative treatment of capacitor discharge is inevitably mathematical. As a capacitor discharges through a resistor, the charge it stores <math>Q</math>, the pd across it <math>V</math>, and the current <math>I</math> in the resistor all decrease exponentially with time. '10% of the maximum value' of the current means 0.1 of the initial current <math>I_0</math>. When the time is to be found from a power of <math>e</math>, taking logs of both sides of the equation provides a solution.</p>
<p>3 (a) Charge stored <math>Q = CV</math>  <math>= 330 \times 10^{-6} \times 9.0 = 2.97 \times 10^{-3} \text{ C}</math>                      Energy stored <math>E = \frac{1}{2} QV</math>  <math>= \frac{1}{2} \times 2.97 \times 10^{-3} \times 9.0 = 1.34 \times 10^{-2} \text{ J}</math></p> <p>(b) Time constant = <math>RC</math>  <math>= 470 \times 10^3 \times 330 \times 10^{-6} = 155 \text{ s}</math></p> <p>(c) When <math>t = 60\text{s}</math>, <math>Q = Q_0 e^{-t/RC}</math> gives  <math>Q = 2.97 \times 10^{-3} e^{-60/155}</math>  <math>= 2.02 \times 10^{-3} \text{ C}</math>                      pd across capacitor <math>V = \frac{Q}{C} = \frac{2.02 \times 10^{-3}}{330 \times 10^{-6}}</math>  <math>= 6.11 \text{ V}</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p><i>Alternatively:</i> energy stored <math>E = \frac{1}{2} CV^2 = \frac{1}{2} \times 330 \times 10^{-6} \times 9.0^2 = 1.34 \times 10^{-2} \text{ J}</math></p> <p>A large value for the time constant indicates that the discharge will be slow.</p> <p>Part (c) is a good test of your ability to use a calculator to find an exponential quantity.</p> <p><i>Alternative approach:</i> using the equation established in Question 2 (c)(ii) above,  <math>V = V_0 e^{-t/RC}</math> gives  <math>V = 9.0 e^{-60/155} = 6.11 \text{ V}</math></p>
<p>4 (a) Use of <math>Q = CV</math> gives                      charge stored <math>Q = 4.7 \times 10^{-6} \times 6.0</math>  <math>= 2.82 \times 10^{-5} \text{ C}</math> or <math>2.8 \times 10^{-5} \text{ C}</math></p> <p>(b) Use of <math>E = \frac{1}{2} CV^2</math>                      From graph, when <math>t = 35 \text{ ms}</math>, <math>V = 2.0 \text{ V}</math>  <math>\therefore E = \frac{1}{2} \times 4.7 \times 10^{-6} \times 2.0^2</math>  <math>= 9.4 \times 10^{-6} \text{ J}</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>The initial charge stored is when time <math>t = 0</math>; at this point on the graph the pd is 6.0 V.</p> <p><i>Alternatively:</i>                      Charge stored  <math>Q = CV = 4.7 \times 10^{-6} \times 2.0 = 9.4 \times 10^{-6} \text{ C}</math>  <math>E = \frac{1}{2} QV = \frac{1}{2} \times 9.4 \times 10^{-6} \times 2.0 = 9.4 \times 10^{-6} \text{ J}</math></p>

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<p>(c) The time constant is the time taken for <math>V</math> to decrease from <math>V_0</math> to <math>(V_0/e)</math>  <math>\therefore V</math> must fall to <math>(6.0/e) = 2.2</math> V                      From the graph, <math>V = 2.2</math> V when <math>t = 32</math> ms  <math>\therefore</math> time constant = 32 ms                      (The graph shows a value between 32 and 33 ms)</p>	<p><b>1</b> <b>1</b> <b>1</b></p>	<p>Other solutions are possible, but all of them are less direct, for example using  <math>V = V_0 e^{-t/RC}</math> for <math>t = 35</math> ms, when  <math>V = 2.0</math> V, gives <math>2.0 = 6.0 e^{-35/RC}</math>  <math>\therefore e^{35/RC} = 3.0</math>                      taking logs, <math>\frac{35}{RC} = \ln 3.0</math>  <math>\therefore</math> time constant  <math>RC = \frac{35}{\ln 3.0} = 32</math> ms</p>
<p>(d) Time constant = <math>RC</math>  <math>\therefore</math> resistance <math>R = \frac{32 \times 10^{-3}}{C} = \frac{32 \times 10^{-3}}{4.7 \times 10^{-3}}</math>  <math>= 6.81 \times 10^3 \Omega</math> or 6.8 k<math>\Omega</math></p>	<p><b>1</b> <b>1</b></p>	<p>This question shows how you might find the resistance of a resistor from a data logging experiment, using the discharge of a capacitor of known capacitance.</p>
<p>5 (a) Time constant = <math>RC</math>  <math>\therefore</math> capacitance of <math>C = \frac{2.2 \times 10^{-4}}{R}</math>  <math>= \frac{2.2 \times 10^{-4}}{220} = 1.0 \times 10^{-6}</math> F (1.0 <math>\mu</math>F)</p>	<p><b>1</b></p>	<p>This circuit needs to have a very small time constant, so that it can be assumed that the capacitor discharges fully whilst the switch is touching contact <b>Y</b>. The farad is a very large unit, so practical capacitors usually have their values marked in <math>\mu</math>F, nF or pF.</p>
<p>(b) Periodic time of oscillation of switch  <math>T = \frac{1}{f} = \frac{1}{400}</math>  <math>= 2.5 \times 10^{-3}</math> s (2.5 ms)</p>	<p><b>1</b> <b>1</b></p>	<p>The question states that the switch touches each contact (<b>X</b> and <b>Y</b>) 400 times per second. It therefore oscillates at a frequency of 400 Hz.</p>
<p>(c) (i) Switch makes contact with <b>Y</b> for a time <math>t = \frac{T}{2} = 1.25 \times 10^{-3}</math> s (1.25 ms) and the time constant of the circuit is 0.22 ms  <math>V = V_0 e^{-t/RC}</math> gives <math>V = 12 e^{-1.25/0.22}</math>  <math>\therefore</math> pd across capacitor at time <math>T/2</math>  <math>V = 4.09 \times 10^{-2}</math> V (40.9 or 41 mV)</p> <p>(ii) It is reasonable to assume that the capacitor has completely discharged in time <math>T/2</math> because <b>either</b></p> <ul style="list-style-type: none"> <li>• 40.9 mV is only 0.34% of 12 V, <b>or</b></li> <li>• 1.25 ms is almost 6 time constants</li> </ul>	<p><b>1</b> <b>1</b> <b>1</b> <b>1</b> <b>1</b></p>	<p>Less direct routes to this answer would be by making use of  <math>Q = Q_0 e^{-t/RC}</math> in combination with <math>Q = CV</math>, <b>or</b> <math>I = I_0 e^{-t/RC}</math> in combination with <math>V = IR</math>. During the discharge, the pd across the capacitor is always equal to the pd across the resistor.</p> <p>Your answer must be accompanied by reasoning to gain the mark. Either of these reasons would be sufficient. Note that in 6 time constants the pd would fall to <math>(\frac{1}{e})^6 = 2.97 \times 10^{-2}</math> V.</p>
<p>6 (a) A capacitor has a capacitance of 1 farad if it stores 1 coulomb of charge when the pd across it is 1 V</p>	<p><b>1</b></p>	<p><math>C = \frac{Q}{V} \therefore 1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}</math></p>

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<p>(b) (i) Charge stored <math>Q = CV</math>  <math>= 2.3 \times 10^{-11} \times 6.0</math>  <math>= 1.38 \times 10^{-10} \text{ C}</math> or  <math>1.4 \times 10^{-10} \text{ C}</math></p>	1 1	This 23 pF capacitor stores only 0.138 nC of charge when the pd across it is 6.0 V.
<p>(ii) Energy stored <math>E = \frac{1}{2}CV^2</math>  <math>= \frac{1}{2} \times 2.3 \times 10^{-11} \times 6.0^2</math>  <math>= 4.14 \times 10^{-10} \text{ J}</math></p>	1 1	Alternatively: $E = \frac{1}{2}QV = \frac{1}{2} \times 1.38 \times 10^{-10} \times 6.0$ $= 4.14 \times 10^{-10} \text{ J}$
<p>(c) Initially (at <math>t = 0</math>), <math>V = 6.0 \text{ V}</math>  when <math>t = 0.4 \text{ ms}</math>, <math>V = 0.80 \text{ V}</math>  use of <math>V = V_0 e^{-t/RC}</math>  gives <math>0.80 = 6.0 e^{-0.0004/RC}</math>  <math>\therefore e^{0.0004/RC} = \frac{6.0}{0.80} = 7.5</math>  taking logs, <math>\frac{0.0004}{RC} \ln 7.5 = 2.01</math>  giving <math>RC = 1.99 \times 10^{-4} \text{ s}</math>  Resistance of oscilloscope  <math>R = \frac{1.99 \times 10^{-4}}{2.3 \times 10^{-11}} = 8.63 \times 10^6 \Omega</math>  or <math>8.6 \times 10^6 \Omega</math></p>	1 1 1 1	<p>These readings are taken from the calibrated screen of the oscilloscope shown in the diagram.</p> <p>Alternatively (and more simply):</p> <p>Since <math>\frac{1}{e} = 0.368</math>, the pd will fall to 0.37 of 6.0 V in a time of 1 time constant (<math>RC</math>).</p> <p>On the graph, <math>V = 2.2 \text{ V}</math> when <math>t = 0.20 \text{ ms}</math>, so <math>RC = 0.20 \text{ ms}</math>. This result gives <math>R = 8.70 \times 10^6 \Omega</math>. Some tolerance has to be allowed in the answers: 8.1 to 8.7 M<math>\Omega</math> should be acceptable.</p>
<p>7 (a) (i) Energy stored <math>E = \frac{1}{2}CV^2</math>  <math>= \frac{1}{2} \times 270 \times 10^{-6} \times 3.0^2</math>  <math>= 1.22 \times 10^{-3} \text{ J}</math> or <math>1.2 \times 10^{-3} \text{ J}</math></p>	1 1	When a capacitor is charged from a battery of emf $V$ so that it stores charge $Q$ , the battery moves a total charge $Q$ across a pd of $V$ and does work $QV$ . Half of this work is lost as thermal energy as the charge flows through resistive components in the circuit.
<p>(ii) Work done by battery = <math>QV</math>  <math>= 2 \times (\text{energy stored by capacitor})</math>  <math>= 2 \times 1.215 \times 10^{-3} = 2.43 \times 10^{-3} \text{ J}</math>  or <math>2.4 \times 10^{-3} \text{ J}</math></p>	1	
<p>(b) (i) When <math>V = 0.3 \text{ V}</math>, energy stored  <math>E = \frac{1}{2}CV^2 = \frac{1}{2} \times 270 \times 10^{-6} \times 0.3^2</math>  <math>= 1.22 \times 10^{-5} \text{ J}</math>  <math>= 0.01</math> of the initial energy <math>E_0</math>  <math>\therefore</math> energy released by capacitor = <math>0.99 E_0</math>, which is almost all of the initial energy</p>	1 1	Alternatively: $E = \frac{1}{2}CV^2$ means that $E \propto V^2$ . In this case, $V$ decreases to 0.1 (or 10%) of its initial value, so $E$ decreases to 0.01 (or 1%) of its initial value. Therefore 99% of the initial energy is released.
<p>(ii) Time constant <math>RC = 1.5 \times 270 \times 10^{-6}</math>  <math>= 4.05 \times 10^{-4} \text{ s}</math>  use of <math>V = V_0 e^{-t/RC}</math> gives <math>2.0 = 3.0 e^{-t/RC}</math>  <math>t = RC \ln \frac{3.0}{2.0} = 4.05 \times 10^{-4} \times 0.405</math>  <math>\therefore</math> duration of light flash <math>t = 1.64 \times 10^{-4} \text{ s}</math>  (0.164 or 0.16 ms)</p>	1 1 1	The torch bulb gives out light from $t = 0$ (when $V = 3.0 \text{ V}$ ) until the time when $V$ has fallen to 2.0 V. This time is calculated by solving the exponential decay equation, involving the use of logs once more.

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<p>(iii) Energy of one photon  <math>= \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{500 \times 10^{-9}}</math>  <math>= 3.98 \times 10^{-19} \text{ J}</math>                      Energy released when capacitor discharges from 3.0 V to 2.0 V  <math>= 1.22 \times 10^{-3} - (\frac{1}{2} \times 270 \times 10^{-6} \times 2.0^2)</math>  <math>= 6.80 \times 10^{-4} \text{ J}</math>                      Number of photons released  <math>= \frac{6.80 \times 10^{-4}}{3.98 \times 10^{-19}} = 1.71 \times 10^{15}</math>                      or <math>1.7 \times 10^{15}</math></p>	<p>1 1 1</p>	<p>The final part of the question tests whether you can remember how to calculate the energy of a photon (covered in <i>AS Physics</i> Unit 1).                      A wavelength of 500 nm corresponds to the average wavelength of visible light. The conclusion of the calculation is that the tiny amount of energy released as the capacitor discharges will produce an incredibly large number of photons.</p>
<p>8 (a) Time constant of circuit <math>RC</math>  <math>= 680 \times 2.2 \times 10^{-6} = 1.50 \times 10^{-3} \text{ s}</math>                      Use of <math>V = V_0 e^{-t/RC}</math> gives <math>2.2 = 5.0 e^{-t/RC}</math>                      from which <math>t = RC \ln \frac{5.0}{2.2}</math>  <math>= 1.50 \times 10^{-3} \times 0.821</math>  <math>\therefore</math> time of contact <math>t = 1.23 \times 10^{-3} \text{ s}</math> (1.23 ms)</p>	<p>1 1 1</p>	<p>This question is an example of the practical use of a capacitor–resistor discharge circuit to measure a very short time. The time for which the metal ball is in contact with the metal block would be much too short to be measured directly. Charge flows from the capacitor and through the resistor whilst the circuit is complete: this only happens during the time when the ball and block make contact.</p>
<p>(b) (i) Initial energy stored <math>E_1 = \frac{1}{2}CV^2</math>  <math>= \frac{1}{2} \times 2.2 \times 10^{-6} \times 5.0^2</math>  <math>= 2.75 \times 10^{-5} \text{ J}</math>                      Final energy stored  <math>E_2 = \frac{1}{2} \times 2.2 \times 10^{-6} \times 2.2^2</math>  <math>= 5.32 \times 10^{-6} \text{ J}</math>                      energy lost by capacitor <math>= E_1 - E_2</math>  <math>= 2.22 \times 10^{-5} \text{ J}</math></p>	<p>1 1</p>	<p>The steps in this calculation could be combined into one expression:  <math>\Delta E = E_1 - E_2</math>  <math>= \frac{1}{2}C(V_1^2 - V_2^2)</math>  <math>= \frac{1}{2} \times 2.2 \times 10^{-6} \times (5.0^2 - 2.2^2)</math>  <math>= 2.22 \times 10^{-5} \text{ J}</math></p>
<p>(ii) The energy is dissipated in the 680 <math>\Omega</math> resistor.                      This energy becomes internal energy of the resistor, and eventually internal (thermal) energy in the surroundings.</p>	<p>1 1</p>	<p>Whenever charge flows there is a current. A current in a resistor causes its internal energy (and therefore its temperature) to increase. When its temperature is higher than the surroundings, the resistor passes energy away, raising the internal energy of the air around it.</p>

Nelson Thornes is responsible for the solution(s) given and they may not constitute the only possible solution(s).